

# Medium effects in the $\Lambda K^+$ pair production by 2.83 GeV protons on nuclei

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## Abstract

We study the  $\Lambda K^+$  pair production in the interaction of protons of 2.83 GeV kinetic energy with C, Cu, Ag, and Au target nuclei in the framework of the nuclear spectral function approach for incoherent primary proton–nucleon, secondary pion–nucleon production processes and processes associated with the creation of intermediate  $\Sigma^0 K^+$  pairs. The approach accounts for the initial proton, final  $\Lambda$  hyperon and  $K^+$  meson absorption in nuclei, target nucleon binding and Fermi motion as well as nuclear mean-field potential effects on these processes. We calculate the  $\Lambda$  momentum dependence of the absolute  $\Lambda K^+$  yield from the considered target nuclei in the kinematical conditions of the ANKE experiment, performed at COSY, within the different scenarios for the  $\Lambda$ -nucleus optical potential. We show that the above observable is appreciable sensitive to this potential in the low-momentum region. Therefore, it can help to determine the  $\Lambda$  hyperon optical potential at finite momenta from the direct comparison of the results of our calculations with the data from the ANKE-at-COSY experiment. We also demonstrate that the two-step pion–nucleon production channels dominate in the low-momentum  $\Lambda K^+$  production in the chosen kinematics and, therefore, they have to be taken into account in the analysis of these data.

# 1. Introduction

The production of strangeness carrying particles has been intensively investigated during long time in many experiments using a variety of beams, targets and energies. Strange particles produced in proton-nucleus and nucleus-nucleus reactions interact with the hadronic environment not only by collisions but also by potential interaction which is expected lead to a change the particle properties in matter. Numerous experiments performed during the past two decades had been aimed at studying the properties of the kaons, antikaons and hyperons in a strongly interacting matter. They were motivated by the theoretically predicted phenomena of the partial restoration of chiral symmetry in hot/dense nuclear matter and possible existence of an antikaon condensate or a presence of hyperons in the dense core of neutron stars (see [1] for a recent review).

The in-medium modification effects on the strange meson properties, mass and width, at nuclear saturation density can be described in terms of complex nuclear optical potential. It was established that the real parts of the  $K^+$  and  $K^0$  repulsive nuclear potentials amounts to 20–40 MeV at normal nuclear density  $\rho_0 = 0.16 \text{ fm}^{-3}$  [2], while the attractive  $K^-$  potential is more strong, although there is no common agreement about its strength so far. The calculations, based on chiral Lagrangians [3,4,5] or on meson-exchange potentials [6,7], predict relatively shallow low-energy  $K^-$ -nucleus potential with central depth of the order of -50 to -80 MeV. On the other hand, fits to the  $K^-$  atomic data in terms of phenomenological density-dependent optical potential or relativistic mean-field calculations [8] lead to much stronger potential with depth of about -200 MeV at density  $\rho_0$ . The reported results of different measurements indicate that the values of the central depth of the potential are spread out over the wide range from -30 to -200 MeV [9]. The recent data on the kaon pair production by protons off nuclei obtained by the ANKE Collaboration do not favor a deep antikaon potential [10]. The imaginary part of the nuclear optical potential, which is responsible for the absorption of the strange mesons with both open ( $K^+$ ,  $K^-$ ) and hidden ( $\phi$ ) strangeness during their way out of nuclei, has been also studied by measuring the so-called transparency ratio [11]. These investigations provide the information on the in-medium meson width or on the in-medium meson-nucleon cross section.

While essential progress has been made in studying the properties of strange mesons in nuclear matter (see [12] for a review), much less attention has been paid to study the properties of the strange baryons in the surrounding nuclear environment. The large series of results on the  $\Lambda$  and  $\Sigma$  hyperon production in proton-nucleus collisions have been compiled in high energy region from 9 to 400 GeV [13], while the experimental information on their creation at lower proton beam energies is very poor although the search for medium modification effects here, in particular, in the near-threshold energy domain looks quite promising. So far, the only experiment here aimed at the study of the  $\Lambda(1115)$  hyperon production in  $p+\text{Nb}$  reaction at proton beam energy of 3.5 GeV has been performed by the HADES Collaboration at SIS18/GSI Darmstadt [14]. This kinetic energy corresponds to an excess energy with respect to the threshold of  $\Lambda(1115)$  production in  $NN$  collisions of 0.63 GeV and to a typical hyperon momentum of about 0.4 GeV/c. However, the investigation of the possible medium effects has been out of the scope of the performed data analysis directed to the studying the most important features of the hyperon production. Rich hypernuclear experimental information [15] enables to study the hyperon-nucleus interaction at (almost) zero hyperon momentum with respect to the nuclear matter.

The in-medium properties of the hyperons at finite momentum, density and temperature have become matter of intense theoretical investigation over the past years. Thus, the medium modification of the  $\Lambda(1520)$  hyperon has been studied within chiral unitary theory with coupled channels [16]. It was found that at normal nuclear matter density the mass shift of the  $\Lambda(1520)$  is small (about 2%), while its in-medium width is more than five times bigger than the free one. The impact of the in-medium  $\Lambda(1520)$  width on the hyperon yield from photon- and proton-nucleus reactions has been analyzed in the frame of collision model based on eikonal approximation in

[17]. The spectral functions for the hyperons have been evaluated in a self-consistent and covariant many-body approach [18]. Attractive mass shifts of about -30 and -40 MeV/c<sup>2</sup> have been predicted for the  $\Lambda(1405)$  and  $\Sigma(1385)$  hyperons at rest at normal nuclear matter density, respectively. It was also found that the  $\Lambda(1115)$  hyperon downward mass shift of about -30 MeV/c<sup>2</sup> at this density is quite independent of the three momentum, but its in-medium width is significantly increased as hyperon moves with respect to the bulk matter. The predicted mass shift for the  $\Sigma(1195)$  is about -22 MeV/c<sup>2</sup> at saturation density. The in-medium properties of the  $\Lambda(1115)$ ,  $\Sigma(1195)$  and  $\Sigma(1385)$  hyperons have been investigated in the chiral unitary approach [19]. It has been found that at the  $\Lambda(1115)$ ,  $\Sigma(1195)$  and  $\Sigma(1385)$  experience an attractive potential of about -50, -40 and -10 MeV, respectively, at normal nuclear matter density and at zero momentum relative to the surrounding nuclear matter. The imaginary parts of the optical potentials of  $\Lambda(1115)$  and  $\Sigma(1195)$  hyperons are less in absolute magnitude than 10 MeV, while that for the  $\Sigma(1385)$  hyperon amounts to -50 MeV at these density and momentum. In contrast to [18], both the real and imaginary parts of the optical potential for all considered hyperons significantly change in the momentum range from 0 to 600 MeV/c. Recently the momentum dependences of the real and imaginary parts of the single-particle potential of the  $\Lambda(1115)$  and  $\Sigma(1195)$  hyperons in isospin symmetric and asymmetric nuclear matter have been studied in chiral effective field theory [20]. At saturation density, the real part of the attractive  $\Lambda(1115)$  potential is between -(24-28) MeV at zero momentum with respect to the nuclear matter, increases with the hyperon momentum and becomes positive (repulsive) at the momenta exceeding 400 MeV/c, while a  $\Sigma(1195)$  feels a repulsive potential even at zero momentum. Cited in [18-20] values of the real part of a  $\Lambda(1115)$  optical potential at saturation density and at zero momentum in the nuclear matter rest frame are somewhat different, but comparable to an empirical value of about -30 MeV deduced from the analysis of binding energies of hypernuclei [15]. On the other hand, the situation with, in particular, the  $\Lambda$  hyperon–nucleus potential at finite momenta is still unclear presently in spite of a lot theoretical activity in this field. The experimental information about the momentum dependence of this potential is completely absent now. This information can be deduced from the analysis of the experimental data on the production of  $\Lambda$  hyperons in coincidence with the  $K^+$  mesons in proton collisions with C, Cu, Ag and Au targets at an initial energy of 2.83 GeV, taken recently by the ANKE Collaboration at COSY accelerator. The advantage of such coincident data compared to the inclusive ones is that the lesser number of individual exclusive elementary  $\Lambda$  production channels should be accounted for in their interpretation with the aim of obtaining the information on the  $\Lambda$ –nuclear potential compared to that which has to be involved in the analysis of the inclusive data. That makes such interpretation more clear and allows for a reduction of the theoretical uncertainties associated with the  $\Lambda$  particle production mechanisms.

In this connection, the main goal of the present work is to give the predictions for the absolute yields of  $\Lambda K^+$  pairs from  $pA$  collisions in the kinematical conditions of the ANKE experiment which will be defined below by the equation (37), adopting the collision model, based on the nuclear spectral function, for incoherent one-step and two-step  $\Lambda K^+$  pair creation processes in different scenarios for the lambda–nuclear potential. Direct comparison of these predictions with the expected data from the this experiment will allow one to shed light on the  $\Lambda$  potential in nuclear medium at finite momenta.

## 2. The model

### 2.1. Direct $\Lambda K^+$ production mechanisms

The direct production of  $\Lambda$  hyperons in coincidence with forward going  $K^+$  mesons in the kinematical conditions of the ANKE experiment in  $pA$  collisions at incident energy of 2.83 GeV of our interest can occur in the following  $pp$  and  $pn$  elementary processes with zero, one and two pions

in the final states <sup>1)</sup> :

$$p + p \rightarrow \Lambda + p + K^+, \quad (1)$$

$$\begin{aligned} p + p &\rightarrow \Lambda + p + \pi^0 + K^+, \\ p + p &\rightarrow \Lambda + n + \pi^+ + K^+; \end{aligned} \quad (2)$$

$$\begin{aligned} p + p &\rightarrow \Lambda + p + \pi^0 + \pi^0 + K^+, \\ p + p &\rightarrow \Lambda + p + \pi^+ + \pi^- + K^+, \\ p + p &\rightarrow \Lambda + n + \pi^0 + \pi^+ + K^+; \end{aligned} \quad (3)$$

$$p + n \rightarrow \Lambda + n + K^+, \quad (4)$$

$$\begin{aligned} p + n &\rightarrow \Lambda + n + \pi^0 + K^+, \\ p + n &\rightarrow \Lambda + p + \pi^- + K^+; \end{aligned} \quad (5)$$

$$\begin{aligned} p + n &\rightarrow \Lambda + n + \pi^0 + \pi^0 + K^+, \\ p + n &\rightarrow \Lambda + n + \pi^+ + \pi^- + K^+, \\ p + n &\rightarrow \Lambda + p + \pi^- + \pi^0 + K^+. \end{aligned} \quad (6)$$

Let us now discuss the total cross sections of the reactions (1)–(6), which we will use throughout our calculations of  $\Lambda$  hyperon yields in  $pA$  collisions. The channel  $pp \rightarrow \Lambda p K^+$  has been extensively studied experimentally both earlier—mostly at beam energies  $\geq 2.85$  GeV [21]—and recently—at initial proton energies  $\leq 2.5$  GeV by the COSY-11, COSY-TOF and ANKE Collaborations (see, for example, [22–30]) as well as very recently—at incident proton energy of 3.5 GeV by the HADES Collaboration [31]. The following parametrization of available experimental data for the total cross section of the  $pp \rightarrow \Lambda p K^+$  reaction will be used by us in the present work:

$$\sigma_{pp \rightarrow \Lambda p K^+}(\sqrt{s}, \sqrt{s_{\text{th}}}) = \begin{cases} \frac{A_\Lambda (s - s_{\text{th}})^2}{4m_p^2 + B_\Lambda (s - s_{\text{th}})^2} & \text{for } \sqrt{s} - \sqrt{s_{\text{th}}} > 0.435 \text{ GeV}, \\ \frac{C_\Lambda (\sqrt{s} - \sqrt{s_{\text{th}}})^2}{\left[1 + \sqrt{1 + (\sqrt{s} - \sqrt{s_{\text{th}}})/D_\Lambda}\right]^2} & \text{for } 0 < \sqrt{s} - \sqrt{s_{\text{th}}} \leq 0.435 \text{ GeV}, \end{cases} \quad (7)$$

where  $\sqrt{s}$  is the total  $pp$  center-of-mass energy and  $\sqrt{s_{\text{th}}} = m_\Lambda + m_p + m_{K^+}$  is the threshold energy;  $m_\Lambda$ ,  $m_p$  and  $m_{K^+}$  are the  $\Lambda$  hyperon, proton and  $K^+$  meson free space masses, respectively; and the constants  $A_\Lambda$ ,  $B_\Lambda$ ,  $C_\Lambda$  and  $D_\Lambda$  are given as:

$$A_\Lambda = 122.943 \text{ } \mu\text{b/GeV}^2, \quad B_\Lambda = 2.015/\text{GeV}^2, \quad C_\Lambda = 25740 \text{ } \mu\text{b/GeV}^2, \quad D_\Lambda = 5.203 \cdot 10^{-3} \text{ GeV}. \quad (8)$$

It combines the relevant fit from [32] in the high excess energy region with that given in the work [29] in the low energy interval. In figure 1 the results of calculations by parametrization (7), (8) for the total cross section  $\sigma_{pp \rightarrow \Lambda p K^+}$  of reaction  $pp \rightarrow \Lambda p K^+$  are shown as a solid line together with the world data [21–31] on this cross section. The arrow on this figure indicates the excess energy that corresponds to the beam kinetic energy of 2.83 GeV. One can see that this parametrization describes the energy dependence of the cross section  $\sigma_{pp \rightarrow \Lambda p K^+}$  quite well both at low and high considered

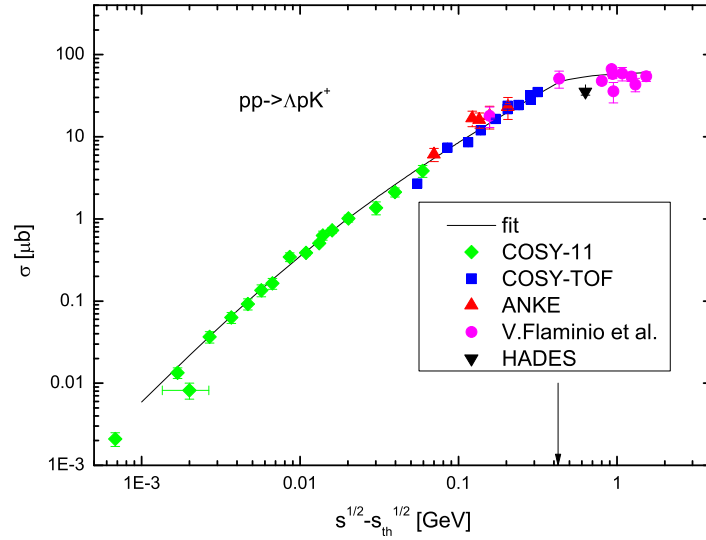


Figure 1: Total cross section for  $pp \rightarrow \Lambda p K^+$  reaction as a function of an excess energy. For notation see the text.

excess energies. Two processes (2) for  $\Lambda$  production together with one pion in  $pp$  interactions as well as the process  $pp \rightarrow \Lambda p \pi^+ K^0$  have been measured at proton energies  $\geq 4.1$  GeV [21], with an exception of a two  $pp \rightarrow \Lambda p \pi^+ K^0$  measurements carried out by the ANKE [33] and HADES [34] Collaborations at 2.83 and 3.5 GeV, respectively. The three sets of data, which are available [21] at beam energies  $\geq 4.1$  GeV for these processes, indicate that they have the similar total cross sections (see, also, [35]):

$$\sigma_{pp \rightarrow \Lambda p \pi^0 K^+} \approx \sigma_{pp \rightarrow \Lambda n \pi^+ K^+} \approx \sigma_{pp \rightarrow \Lambda p \pi^+ K^0}. \quad (9)$$

We assume that the relations (9) among the cross sections  $\sigma_{pp \rightarrow \Lambda N \pi K}$  are valid also at lower incident proton energies. For the free total cross section  $\sigma_{pp \rightarrow \Lambda p \pi^+ K^0}$  we have used the following parametrization:

$$\sigma_{pp \rightarrow \Lambda p \pi^+ K^0}(\sqrt{s}, \sqrt{s_{1th}}) = \begin{cases} 1770.5 (\sqrt{s} - \sqrt{s_{1th}})^{5.62} [\mu b] & \text{for } 0 < \sqrt{s} - \sqrt{s_{1th}} \leq 0.5 \text{ GeV}, \\ 72 (\sqrt{s} - \sqrt{s_{1th}}) [\mu b] & \text{for } \sqrt{s} - \sqrt{s_{1th}} > 0.5 \text{ GeV}, \end{cases} \quad (10)$$

where  $\sqrt{s_{1th}} = m_\Lambda + m_p + m_{\pi^+} + m_{K^0}$  is the threshold energy. Here,  $m_{\pi^+}$  and  $m_{K^0}$  are the  $\pi^+$  and  $K^0$  meson free space masses, respectively. The comparison of the results of calculations by (10) (solid line) with the experimental data for  $pp \rightarrow \Lambda p \pi^+ K^0$  reaction from the installation ANKE [33] (full triangle), from the HADES Collaboration [34] (full square <sup>2)</sup>) as well as with the data [21] at higher energies (full circles) is shown in figure 2. In this figure we also show the predictions from the parametrization (45) (dashed line) employed in the study [35] of kaon creation

<sup>1)</sup>Recall that the free threshold energies, e.g., for the processes  $pp \rightarrow \Lambda p K^+$ ,  $pp \rightarrow \Lambda p \pi^0 K^+$  and  $pp \rightarrow \Lambda p \pi^0 \pi^0 K^+$  amount, respectively, to 1.58, 1.96 and 2.35 GeV. We can neglect at beam energy of interest the subprocesses  $pN \rightarrow \Lambda N 3\pi K$  with three pions in the final states due to the proximity of their production thresholds in free  $pN$  interactions to this energy. Thus, for instance, the threshold energy of the channel  $pp \rightarrow \Lambda p 3\pi^0 K^+$  is 2.77 GeV. This energy is close to the incident proton energy of 2.83 GeV. Hence, the subprocesses  $pN \rightarrow \Lambda N 3\pi K$  are energetically suppressed.

<sup>2)</sup>This data point has been inferred from the measured total cross sections for direct and resonant (via the intermediate  $\Delta^{++}$ ) production of 4-body final state  $\Lambda p \pi^+ K^0$  in  $pp$  interactions.

in heavy-ion collisions. It is seen that this parametrization significantly overestimates the lowest data point, obtained at 2.83 GeV initial proton kinetic energy. Now, we consider total cross sections

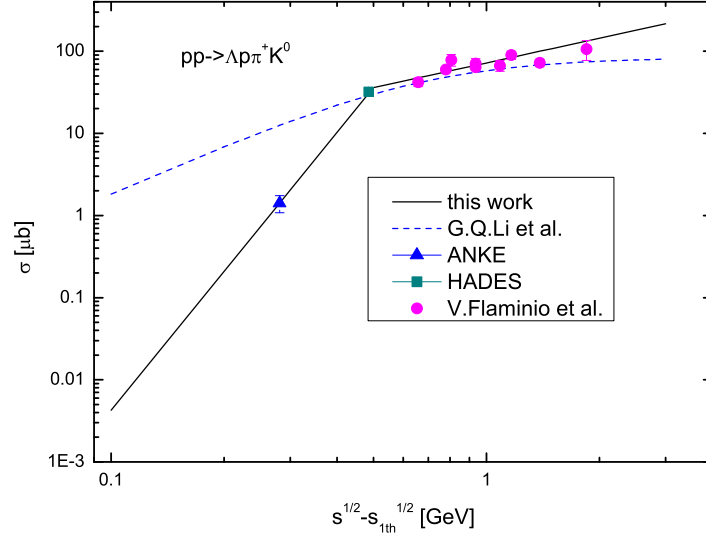


Figure 2: Total cross section for  $pp \rightarrow \Lambda p \pi^+ K^0$  reaction as a function of an excess energy. For notation see the text.

of the reactions (3) with two pions in the final states. The experimental data for these reactions are quite scarce. Until now, there exist the data for the total cross section  $\sigma_{pp \rightarrow \Lambda p \pi^+ \pi^- K^+}$  of only process  $pp \rightarrow \Lambda p \pi^+ \pi^- K^+$  [21]. Data also are available for the total cross sections  $\sigma_{pp \rightarrow \Lambda p \pi^+ \pi^0 K^0}$  and  $\sigma_{pp \rightarrow \Lambda n \pi^+ \pi^+ K^0}$  of reactions  $pp \rightarrow \Lambda p \pi^+ \pi^0 K^0$  and  $pp \rightarrow \Lambda n \pi^+ \pi^+ K^0$  [21]. These data were obtained at beam energies beginning with 4.100, 4.641 and 6.045 GeV, respectively, and they can be approximately fitted by the following expression suggested in [35]:

$$\sigma_{pp \rightarrow \Lambda p \pi^+ \pi^- K^+} \approx \sigma_{pp \rightarrow \Lambda p \pi^+ \pi^0 K^0} \approx \sigma_{pp \rightarrow \Lambda n \pi^+ \pi^+ K^0} \approx \frac{80 \left( \sqrt{s} - \sqrt{s_{2th}} \right)^2}{2.25 + \left( \sqrt{s} - \sqrt{s_{2th}} \right)^2} [\mu\text{b}], \quad (11)$$

where  $\sqrt{s_{2th}} = m_\Lambda + m_p + 2m_{\pi^+} + m_{K^+}$  is the threshold energy. We assume that other two channels  $pp \rightarrow \Lambda p \pi^0 \pi^0 K^+$  and  $pp \rightarrow \Lambda n \pi^+ \pi^0 K^+$  have the same cross section [35]. We will also use the parametrization (11) in the present work at incident energy of 2.83 GeV, which is lower than those studied in the compilation [21] <sup>3)</sup>.

Finally, we focus upon the total cross sections for  $\Lambda$  production in  $pn$  reactions (4)–(6). Up to now, there have been no direct data on  $\Lambda$  production in reaction (4). The relationship between total cross section  $\sigma_{pn \rightarrow \Lambda n K^+}$  of this reaction and that  $\sigma_{pp \rightarrow \Lambda p K^+}$  of channel (1) can be obtained by the following indirect route. An analysis of the data on the production of  $K^+$  mesons at small angles in proton–proton and proton–deuteron collisions at beam energies of 1.826, 1.920, 2.020 and 2.650 GeV, taken by the ANKE Collaboration [37], gave for the ratio of inclusive total  $K^+$  creation

<sup>3)</sup>It should be pointed out that the relations (11) are in line with the results for the total cross sections for  $K^0$  production channels  $pp \rightarrow \Lambda p \pi^+ \pi^0 K^0$  and  $pp \rightarrow \Lambda n \pi^+ \pi^+ K^0$  at beam kinetic energy of 3.5 GeV, which can be obtained assuming that these channels go exclusively through the reactions  $pp \rightarrow \Delta^{++} \Sigma(1385)^0 K^0$  and  $pp \rightarrow \Delta^+ \Sigma(1385)^+ K^0$  and using formula (2) for the total cross sections of latter ones from work [36], in which the inclusive  $K^0$  production in  $pp$  and  $p\text{Nb}$  interactions has been measured with the HADES detector. Therefore, this gives confidence to us that the adoption of these relations is justified at beam energy of our interest.

cross sections in  $pn$  and  $pp$  interactions the value  $\sigma_{pn}^{K^+}/\sigma_{pp}^{K^+} = 0.5 \pm 0.2$  at all the four energies investigated. Since the main contribution to the total cross sections of the reactions  $pn \rightarrow K^+X$  and  $pp \rightarrow K^+X$  comes, at least at initial energies  $\leq 2.020$  GeV, from the channels  $pn \rightarrow \Lambda n K^+$  and  $pp \rightarrow \Lambda p K^+$  [38], this means that

$$\sigma_{pn \rightarrow \Lambda n K^+} \approx \frac{1}{2} \sigma_{pp \rightarrow \Lambda p K^+} \quad (12)$$

at these energies. Further, the  $\Lambda$  hyperon production cross sections in reaction (4) and in channel  $pn \rightarrow \Lambda p K^0$  are the same due to the isospin symmetry. There is [21] only five data points for the free total cross section  $\sigma_{pn \rightarrow \Lambda p K^0}$  of this channel at 5.135, 6.124 and 16.088 GeV incident proton kinetic energies. Comparing the latter ones with the data, which are available [21] for  $pp \rightarrow \Lambda p K^+$  channel at similar energies (at energies of 5.135, 6.045 and 11.098 GeV), one can easily get that at these energies the  $pn$  cross sections  $\sigma_{pn \rightarrow \Lambda p K^0}$  and  $\sigma_{pn \rightarrow \Lambda n K^+}$  are about half of the  $pp$  one  $\sigma_{pp \rightarrow \Lambda p K^+}$ , namely:

$$\sigma_{pn \rightarrow \Lambda p K^0} = \sigma_{pn \rightarrow \Lambda n K^+} \approx \frac{1}{2} \sigma_{pp \rightarrow \Lambda p K^+}. \quad (13)$$

Accounting for the expressions (12) and (13), we will assume that

$$\sigma_{pn \rightarrow \Lambda n K^+}(\sqrt{s}) \approx \frac{1}{2} \sigma_{pp \rightarrow \Lambda p K^+}(\sqrt{s}, \sqrt{s_{\text{th}}}) \quad (14)$$

also at all center-of-mass energies  $\sqrt{s}$ , accessible in calculation of  $\Lambda K^+$  production in  $pA$  reactions at beam energy of 2.83 GeV of our interest with allowance for the Fermi motion of intranuclear nucleons <sup>4)</sup>. The only set of data is available for  $\Lambda$  hyperon production together with one pion in  $pn$  collisions. It was obtained for  $pn \rightarrow \Lambda p \pi^- K^+$  reaction at incident energies beginning with 5.135 GeV [21]. The set indicates the total cross section  $\sigma_{pn \rightarrow \Lambda p \pi^- K^+}$  of this reaction, which is about half of those (9), (10) for  $pp$  processes (2) (cf. [35]). In line with [35], we will assume that other  $pn$  channel  $pn \rightarrow \Lambda n \pi^0 K^+$  from (5) has the same cross section as the  $pn \rightarrow \Lambda p \pi^- K^+$  reaction. Under the above assumptions, we have:

$$\sigma_{pn \rightarrow \Lambda n \pi^0 K^+}(\sqrt{s}) \approx \sigma_{pn \rightarrow \Lambda p \pi^- K^+}(\sqrt{s}) \approx \frac{1}{2} \sigma_{pp \rightarrow \Lambda p \pi^+ K^0}(\sqrt{s}, \sqrt{s_{1\text{th}}}). \quad (15)$$

The relations (15) will be used in our calculations of  $\Lambda$  hyperon yields from  $pA$  collisions at all accessible values of the center-of-mass energy  $\sqrt{s}$ . The only set of data there is presently for  $\Lambda$  hyperon production together with two pions in  $pn$  interactions as well. It was taken for the  $pn \rightarrow \Lambda p \pi^+ \pi^- K^0$  reaction at beam energies of 5.135, 6.124 and 16.088 GeV [21]. The set indicates that the total cross section  $\sigma_{pn \rightarrow \Lambda p \pi^+ \pi^- K^0}$  of this reaction is about half of those (11) for respective  $pp$  channels (3) (cf. [35]). As in [35], we will assume that all three  $pn$  processes (6) have the same cross section as the  $pn \rightarrow \Lambda p \pi^+ \pi^- K^0$  reaction. In the context of above, one gets:

$$\sigma_{pn \rightarrow \Lambda n \pi^0 \pi^0 K^+}(\sqrt{s}) \approx \sigma_{pn \rightarrow \Lambda n \pi^+ \pi^- K^+}(\sqrt{s}) \approx \sigma_{pn \rightarrow \Lambda p \pi^- \pi^0 K^+}(\sqrt{s}) \quad (16)$$

$$\approx \sigma_{pn \rightarrow \Lambda p \pi^+ \pi^- K^0}(\sqrt{s}) \approx \frac{1}{2} \sigma_{pp \rightarrow \Lambda p \pi^+ \pi^- K^+}(\sqrt{s}, \sqrt{s_{2\text{th}}}).$$

We will use the relations (16) at all collision energies of interest. It is worth noting that, as follows from the expressions (7)–(16), the ratios of  $\Lambda$  production cross sections in  $pp$  and  $pn$  processes (2), (3) and (5), (6) with one as well as with two pions in the final states to those of  $\Lambda$  creation

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<sup>4)</sup>It is interesting to note that theoretical estimates of the ratio  $\sigma_{pn \rightarrow \Lambda n K^+}/\sigma_{pp \rightarrow \Lambda p K^+}$ , obtained on the basis of the meson-exchange models [39–41], are around of 2 [39], 3 [40] or range from 0.25 to 10 [41], which is inconsistent with experimental results (12) and (13).

channels (1) and (4) with zero outgoing pions are about of 1/9 at kinetic beam energy of 2.83 GeV of interest. This means that the main contribution to the  $\Lambda$  production in  $pA$  reactions even at this high incident energy comes from the latter 3-body primary channels (1) and (4).

In the following calculations we will include the medium modification of the final nucleon, kaon and  $\Lambda$  hyperon, participating in the production processes (1)–(6), by using in the in-medium cross sections of these processes, for reasons of numerical simplicity, instead of their local effective masses  $m_h^*(r)$  their average in-medium masses  $\langle m_h^* \rangle$  defined in line with [42] as:

$$\langle m_h^* \rangle = m_h + U_h \frac{\langle \rho_N \rangle}{\rho_0}. \quad (17)$$

Here,  $m_h$  is the hadron free space mass,  $U_h$  is the value of its nuclear potential at saturation density  $\rho_0$ ,  $\langle \rho_N \rangle$  is the average nucleon density, which was calculated separately for each considered target nucleus. Our calculations show that for target nuclei C, Cu, Ag and Au the ratio  $\langle \rho_N \rangle / \rho_0$  is approximately equal to 0.55, 0.65, 0.72 and 0.77, respectively. The above values will be used throughout the following study. In it we assume that pions do not change their properties in the nuclear medium at densities of ordinary nuclei [43]. To match smoothly in-medium  $\Lambda$  hyperon production thresholds in  $pp$  collisions to those in  $pn$  interactions, we also neglect the influence of the Coulomb potentials on final charged hadrons (protons, pions and kaons), participating in the elementary reactions (1)–(6). In addition, these potentials, as is expected, have a minor role on the  $\Lambda$  dynamics at initial proton energy of our interest. The total energy  $E'_h$  of the hadron inside the nuclear medium can be expressed through its average effective mass  $\langle m_h^* \rangle$  defined above and its in-medium momentum  $\mathbf{p}'_h$  as in the free particle case, namely:

$$E'_h = \sqrt{(\langle m_h^* \rangle)^2 + (\mathbf{p}'_h)^2}. \quad (18)$$

The momentum  $\mathbf{p}'_h$  is related to the vacuum one  $\mathbf{p}_h$  by the following expression:

$$E'_h = \sqrt{(\langle m_h^* \rangle)^2 + (\mathbf{p}'_h)^2} = \sqrt{m_h^2 + \mathbf{p}_h^2} = E_h, \quad (19)$$

where  $E_h$  is the hadron vacuum total energy. In the subsequent study for the  $K^+$  mass shift  $U_{K^+}$  we will always employ the following option:  $U_{K^+} = 22$  MeV [44]. The same option will be adopted for the  $K^0$  mass shift  $U_{K^0}$ . Accounting for that the relation between the effective scalar nucleon potential  $U_N$ , entering into the equation (17), and the corresponding Schrödinger equivalent potential  $V_{NA}^{\text{SEP}}$  (or the so-called single-particle potential) at the normal nuclear matter density is given by

$$U_N = \frac{\sqrt{m_N^2 + p_N'^2}}{m_N} V_{NA}^{\text{SEP}}, \quad (20)$$

as well as the fact that the vacuum momenta of the outgoing in reactions (1)–(6) nucleons are, as showed our calculations, around of momentum  $p_N \approx 0.6$  GeV/c in the kinematics of ANKE experiment, assuming that  $p'_N \approx p_N$ <sup>5)</sup> and using that  $V_{NA}^{\text{SEP}} \approx 0$  MeV at this in-medium nucleon momentum [45], we can readily obtain that  $U_N \approx 0$  MeV. We will employ this potential throughout our present work.

Let us now specify the effective scalar mean-field  $\Lambda$  hyperon potential  $U_\Lambda$ , entering into the formula (17). A nuclear optical potential acting on the bound in the nucleus low-momentum  $\Lambda$  hyperon has been extracted from the properties of hypernuclei [46, 47]. This potential has been also investigated in the framework of a relativistic mean-field theory [48, 49]. It was obtained from these studies that the well depth for a  $\Lambda$  particle embedded into the nuclear matter is in the vicinity

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<sup>5)</sup>This point has been numerically checked by employing the equation (19), the Schrödinger equivalent potential  $V_{NA}^{\text{SEP}}$  from [45] as well as assuming that  $\langle \rho_N \rangle / \rho_0 = 0.5$ .



of 30 MeV. The kinetic energy dependence of the  $\Lambda$ -nucleus mean-field potential for  $\Lambda$  hyperons colliding with a nucleus was evaluated in [50] within the G-matrix theory over the  $\Lambda$  kinetic energy range of 0–70 MeV (or over the  $\Lambda$  momentum interval of 0–0.4 GeV/c). The momentum dependence of this potential was studied in [51] within the framework of the relativistic Brueckner–Hartree–Fock theory employing two kinds of  $YN$  phenomenological potentials – Juelich 94a and Juelich 05, constructed with the meson-exchange model by the Juelich group, at the same  $\Lambda$  momenta as in [50] and, let us remind, in [19] within respective chiral unitary coupled-channel approach for  $\Lambda$  momenta ranging up to 0.6 GeV/c. In both cases [51] and [19], the similar behavior was found for the  $\Lambda$  optical potential at normal nuclear matter density: it is attractive and monotonically increase with the growth of  $\Lambda$  momentum. The  $\Lambda$  single-particle potential in isospin symmetric and asymmetric nuclear matter at finite momenta up to 0.6 GeV/c has been also investigated recently in [20] in the framework of the Brueckner approach using the  $\Lambda N$  potential derived from SU(3) chiral effective field theory at next-to-leading order as was already noted above. Contrary to the results from [51, 19], this potential turns from attractive to repulsive at about 0.4 GeV/c momentum in isospin symmetric nuclear medium at saturation density. Moreover, the  $\Lambda$  single-particle potential in symmetric nuclear matter has been calculated in [52] within the SU<sub>6</sub> quark model at various nuclear densities as a function of the momentum in the range of about 0–1.4 GeV/c. An essential difference between this potential and that from [20] is that it turns to repulsion at fairly high momenta around 1.1 GeV/c. Since the accessible range of the  $\Lambda$  hyperon momenta in the ANKE experiment is about of 0.4–3 GeV/c, it is helpful to estimate the  $\Lambda$  optical potential, needed for our present study, also for such high  $\Lambda$  momenta. We will rely on the constituent quark model, which has been employed in [53] to derive the density dependence of low-momentum  $\Lambda$ -nucleus potential, and will proceed analogously with the aim of obtaining its momentum dependence at saturation density  $\rho_0$ . In this model, the  $\Lambda$  mean-field scalar  $U_{S\Lambda}$  and vector  $U_{V\Lambda}$  potentials are about 2/3 of those  $U_{SN}$  and  $U_{VN}$  of a nucleon when in-medium nucleon and  $\Lambda$  hyperon velocities  $v'_N$  and  $v'_\Lambda$  relative to the nuclear matter are equal to each other, i.e.,

$$U_{S\Lambda}(v'_\Lambda, \rho_N) = \frac{2}{3}U_{SN}(v'_N, \rho_N), \quad U_{V\Lambda}(v'_\Lambda, \rho_N) = \frac{2}{3}U_{VN}(v'_N, \rho_N); \quad v'_N = v'_\Lambda. \quad (21)$$

The latter in (21) corresponds, as is easy to see, to the following relation between the respective in-medium nucleon momentum  $p'_N$  and the  $\Lambda$  one  $p'_\Lambda$ :

$$p'_N = \frac{\langle m_N^* \rangle}{\langle m_\Lambda^* \rangle} p'_\Lambda. \quad (22)$$

However, for reasons of numerical simplicity, calculating the  $\Lambda$ -nucleus optical potential, we will employ in expression (22) free space nucleon and  $\Lambda$  hyperon masses  $m_N$  and  $m_\Lambda$  instead of their average in-medium masses  $\langle m_N^* \rangle$  and  $\langle m_\Lambda^* \rangle$ . Then, the lambda Schrödinger equivalent potential  $V_{\Lambda A}^{\text{SEP}}$  can be defined as [53]:

$$V_{\Lambda A}^{\text{SEP}}(p'_\Lambda, \rho_N) = \sqrt{[m_\Lambda + U_{S\Lambda}(p'_\Lambda, \rho_N)]^2 + p'^2_\Lambda} + U_{V\Lambda}(p'_\Lambda, \rho_N) - \sqrt{m_\Lambda^2 + p'^2_\Lambda}. \quad (23)$$

Adopting the momentum-dependent parametrization for the nucleon scalar and vector potentials from [54] and using the equations (22), (23), we calculated the momentum dependence of optical potential  $V_{\Lambda A}^{\text{SEP}}$  at normal nuclear matter density  $\rho_0$ . It is shown in figure 3 by dashed curve. We have also made an adjustment by multiplying the vector  $\Lambda$  hyperon potential by a factor of 1.068 to get for potential  $V_{\Lambda A}^{\text{SEP}}$  at zero momentum a value consistent with the experimental one of  $-(32 \pm 2)$  MeV (full circle in figure 3), extracted from data on binding energies of  $\Lambda$  single-particle states in nuclei [46, 47]. The adjusted in such manner potential is presented in figure 3 by solid curve. One can see that in this case the  $\Lambda$ -nucleus potential is attractive for momenta  $\leq 0.7$  GeV/c, whereas it becomes repulsive for higher momenta and reaches the value  $\approx 70$  MeV at  $\Lambda$  momentum of 3 GeV/c.

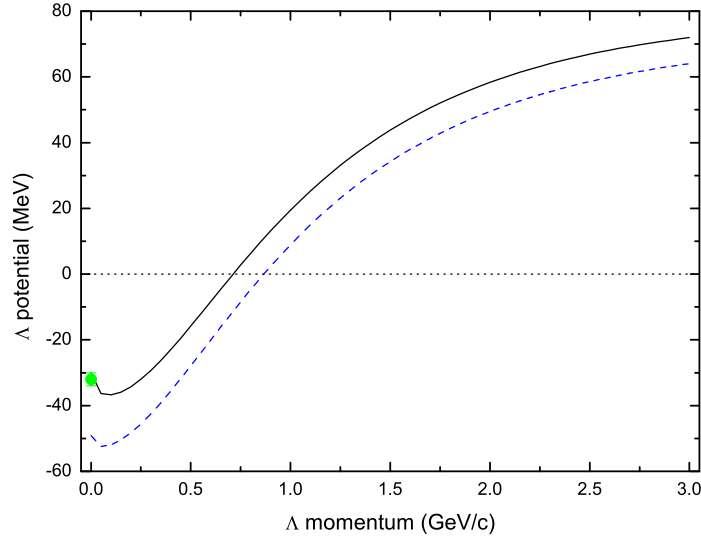


Figure 3: Schrödinger equivalent  $\Lambda$  hyperon potential at density  $\rho_0$  as a function of  $\Lambda$  momentum relative to the nuclear matter at rest calculated on the basis of equation (23) without and with rescaling its vector potential by a factor of 1.068 (dashed and solid lines, respectively).

Taking into account that the relation between the effective scalar hyperon potential  $U_\Lambda$ , entering into the equation (17), and the corresponding Schrödinger equivalent potential  $V_{\Lambda\Lambda}^{\text{SEP}}$  at the normal nuclear matter density is given by the relation analogous to (20) for nucleon as well as the fact that the coincident  $\Lambda K^+$  yield is appreciably sensitive to the lambda potential at vacuum momenta around of momentum  $p_\Lambda \approx 1 \text{ GeV/c}$ <sup>6)</sup>, where the secondary pion–nucleon  $\Lambda K^+$  production channels (39) and (40) considered below are dominant, assuming that, as in nucleon case above,  $p'_\Lambda \approx p_\Lambda$  and using, as follows from figure 3, that  $V_{\Lambda\Lambda}^{\text{SEP}} \approx 20 \text{ MeV}$  for this in-medium lambda momentum, we can readily obtain that  $U_\Lambda \approx 30 \text{ MeV}$ . Since the  $\Lambda K^+$  yield from the considered direct  $\Lambda K^+$  production mechanisms concentrates mainly at  $\Lambda$  momenta around of momentum  $p_\Lambda \approx 2.6 \text{ GeV/c}$  for which it reacts weakly on the  $\Lambda$ –nuclear potential<sup>7)</sup>, we will adopt the value  $U_\Lambda \approx 30 \text{ MeV}$  also in our calculations of this yield. However, in reality it is unclear presently which  $\Lambda$ –nucleus potential is the correct one for such high  $\Lambda$  momenta (cf., for instance, results from Refs. [20] and [52]). Therefore, to extend the range of applicability of our model we will perform the calculations of the  $\Lambda$  production cross sections off nuclei also in the scenarios with possible  $\Lambda$  mass shift  $U_\Lambda$  ranging from -30 MeV to 60 MeV.

Accounting for the distortions of the initial proton and final  $\Lambda$  hyperon and kaon as well as the fact that in the ANKE experiment the latter were detected in the forward polar angular domains  $0^\circ \leq \theta_\Lambda \leq 6^\circ$  and  $0^\circ \leq \theta_{K^+} \leq 12^\circ$ , respectively, and using the results given in [55–58], we can represent the differential cross section for the production on nuclei  $\Lambda$  hyperon with the vacuum momentum  $\mathbf{p}_\Lambda$  in coincidence with the  $K^+$  meson with the vacuum momentum  $\mathbf{p}_{K^+}$  in the primary proton-induced reaction channels (1)–(6) as follows:

$$\frac{d\sigma_{pA \rightarrow \Lambda K^+ X}^{(\text{prim})}(\mathbf{p}_0, \mathbf{p}_\Lambda, \mathbf{p}_{K^+})}{d\mathbf{p}_\Lambda d\mathbf{p}_{K^+}} = I_{K^+\Lambda}[A] \quad (24)$$

<sup>6)</sup>See figures 5–10 given below.

<sup>7)</sup>See figures 7–10 presented below.

$$\times \left[ \frac{Z}{A} \left\langle \frac{d\sigma_{pp \rightarrow \Lambda K^+ X}(\mathbf{p}'_0, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+})}{d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}} \right\rangle_A + \frac{N}{A} \left\langle \frac{d\sigma_{pn \rightarrow \Lambda K^+ X}(\mathbf{p}'_0, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+})}{d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}} \right\rangle_A \right] \frac{d\mathbf{p}'_\Lambda}{d\mathbf{p}_\Lambda} \frac{d\mathbf{p}'_{K^+}}{d\mathbf{p}_{K^+}},$$

where

$$I_{K^+\Lambda}[A] = 2\pi A \int_0^R r_\perp dr_\perp \int_{-\sqrt{R^2-r_\perp^2}}^{\sqrt{R^2-r_\perp^2}} dz \rho(\sqrt{r_\perp^2 + z^2}) \quad (25)$$

$$\begin{aligned} & \times \exp \left[ -\sigma_{pN}^{\text{in}} A \int_{-\sqrt{R^2-r_\perp^2}}^z \rho(\sqrt{r_\perp^2 + x^2}) dx - \sigma_{K^+N}^{\text{tot}} A \int_z^{\sqrt{R^2-r_\perp^2}} \rho(\sqrt{r_\perp^2 + x^2}) dx \right] \\ & \times \exp \left[ -\sigma_{\Lambda N}^{\text{tot}}(p'_\Lambda) A \int_z^{\sqrt{R^2-r_\perp^2}} \rho(\sqrt{r_\perp^2 + x^2}) dx \right], \\ & \left\langle \frac{d\sigma_{pN \rightarrow \Lambda K^+ X}(\mathbf{p}'_0, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+})}{d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}} \right\rangle_A = \int \int P_A(\mathbf{p}_t, E) d\mathbf{p}_t dE \\ & \times \left\{ \frac{d\sigma_{pN \rightarrow \Lambda K^+ X}[\sqrt{s}, < m_{K^+}^* >, < m_N^* >, < m_\Lambda^* >, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+}]}{d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}} \right\} \end{aligned} \quad (26)$$

and

$$p'_\Lambda = \sqrt{E_\Lambda^2 - (< m_\Lambda^* >)^2}, \quad E_\Lambda = \sqrt{m_\Lambda^2 + \mathbf{p}_\Lambda^2}. \quad (27)$$

Here,  $d\sigma_{pN \rightarrow \Lambda K^+ X}[\sqrt{s}, < m_{K^+}^* >, < m_N^* >, < m_\Lambda^* >, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+}]/d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}$  are the "in-medium" differential cross sections for the production of  $\Lambda$  hyperon and  $K^+$  meson with the in-medium momenta  $\mathbf{p}'_\Lambda$  and  $\mathbf{p}'_{K^+}$ , correspondingly, in reactions (1)–(3) ( $N = p$ ) and in (4)–(6) ( $N = n$ ) at the  $pN$  center-of-mass energy  $\sqrt{s}$ ;  $\rho(r)$  and  $P_A(\mathbf{p}_t, E)$  are the local nucleon density and the spectral function of target nucleus  $A$  normalized to unity;  $\mathbf{p}_t$  and  $E$  are the internal momentum and binding energy of the struck target nucleon just before the collision;  $\sigma_{pN}^{\text{in}}$  and  $\sigma_{K^+N}^{\text{tot}}$ ,  $\sigma_{\Lambda N}^{\text{tot}}$  are the inelastic and total cross sections of the  $pN$  and  $K^+N$ ,  $\Lambda N$  interactions;  $Z$  and  $N$  are the numbers of protons and neutrons in the target nucleus ( $A = Z + N$ ),  $R$  is its radius;  $\mathbf{p}_0$  and  $\mathbf{p}'_0$  are the momenta of the initial proton outside and inside the target nucleus. They are linked by the equation (10) from [57]. The expression for  $s$  is given in [57] by the formula (8). The specific information about the quantities  $\rho(r)$  and  $P_A(\mathbf{p}_t, E)$ , adopted in our subsequent calculations, is given in [42, 44, 59]. Using the total cross sections  $\sigma_{K^+N}^{\text{tot}}$  and  $\sigma_{\Lambda N}^{\text{tot}}$  in (25), we assume that if a kaon or a lambda hyperon undergoes a quasi-elastic collision with the target nucleon it will not fall in the ANKE acceptance window.

Taking into account the above arguments that the main contribution to the primary  $\Lambda K^+$  production in  $pA$  collisions at considered beam energy of 2.83 GeV will come from the three-body direct processes (1) and (4), we describe the in-medium differential cross sections  $d\sigma_{pN \rightarrow \Lambda K^+ X}[\sqrt{s}, < m_{K^+}^* >, < m_N^* >, < m_\Lambda^* >, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+}]/d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}$  according to the three-body phase space. Following [57], we obtain:

$$\begin{aligned} & \frac{d\sigma_{pN \rightarrow \Lambda K^+ X}[\sqrt{s}, < m_{K^+}^* >, < m_N^* >, < m_\Lambda^* >, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+}]}{d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}} = \frac{1}{8E'_\Lambda E'_{K^+}} \\ & \times \frac{\sigma_{pN \rightarrow \Lambda K^+ X}(\sqrt{s}, \sqrt{s_{\text{th}}^*}, \sqrt{s_{\text{1th}}^*}, \sqrt{s_{\text{2th}}^*})}{I_3(s, < m_{K^+}^* >, < m_N^* >, < m_\Lambda^* >)} \frac{1}{(\omega + E_t)} \delta \left( \omega + E_t - \sqrt{(< m_N^* >)^2 + (\mathbf{Q} + \mathbf{p}_t)^2} \right), \end{aligned} \quad (28)$$

where

$$\omega = E'_0 - E'_\Lambda - E'_{K^+}, \quad \mathbf{Q} = \mathbf{p}'_0 - \mathbf{p}'_\Lambda - \mathbf{p}'_{K^+} \quad (29)$$

and the quantities  $E'_0$ ,  $E_t$ ,  $I_3$  are defined in [57] by the equations (9), (12), (14), respectively. Here,  $\sigma_{pN \rightarrow \Lambda K^+ X}(\sqrt{s}, \sqrt{s_{\text{th}}^*}, \sqrt{s_{1\text{th}}^*}, \sqrt{s_{2\text{th}}^*})$  are the "in-medium" total cross sections for the  $\Lambda K^+$  production in reactions (1)–(3) ( $N = p$ ) as well as in (4)–(6) ( $N = n$ ) having the threshold energies  $\sqrt{s_{\text{th}}^*} = \langle m_\Lambda^* \rangle + \langle m_p^* \rangle + \langle m_{K^+}^* \rangle$ ,  $\sqrt{s_{1\text{th}}^*} = \langle m_\Lambda^* \rangle + \langle m_p^* \rangle + m_{\pi^+} + \langle m_{K^0}^* \rangle$  and  $\sqrt{s_{2\text{th}}^*} = \langle m_\Lambda^* \rangle + \langle m_p^* \rangle + 2m_{\pi^+} + \langle m_{K^+}^* \rangle$ . As in [58], we assume that these cross sections are equivalent to the vacuum ones  $\sigma_{pN \rightarrow \Lambda K^+ X}(\sqrt{s}, \sqrt{s_{\text{th}}}, \sqrt{s_{1\text{th}}}, \sqrt{s_{2\text{th}}})$  in which the free threshold energies  $\sqrt{s_{\text{th}}}$ ,  $\sqrt{s_{1\text{th}}}$  and  $\sqrt{s_{2\text{th}}}$  are replaced by the in-medium threshold energies  $\sqrt{s_{\text{th}}^*}$ ,  $\sqrt{s_{1\text{th}}^*}$  and  $\sqrt{s_{2\text{th}}^*}$ . Due to the above considerations, the vacuum cross sections  $\sigma_{pp \rightarrow \Lambda K^+ X}(\sqrt{s}, \sqrt{s_{\text{th}}}, \sqrt{s_{1\text{th}}}, \sqrt{s_{2\text{th}}})$  and  $\sigma_{pn \rightarrow \Lambda K^+ X}(\sqrt{s}, \sqrt{s_{\text{th}}}, \sqrt{s_{1\text{th}}}, \sqrt{s_{2\text{th}}})$  can be defined as:

$$\sigma_{pp \rightarrow \Lambda K^+ X}(\sqrt{s}, \sqrt{s_{\text{th}}}, \sqrt{s_{1\text{th}}}, \sqrt{s_{2\text{th}}}) = \sigma_{pp \rightarrow \Lambda p K^+}(\sqrt{s}, \sqrt{s_{\text{th}}}) + 2\sigma_{pp \rightarrow \Lambda p \pi^+ K^0}(\sqrt{s}, \sqrt{s_{1\text{th}}}) \quad (30)$$

$$+ 3\sigma_{pp \rightarrow \Lambda p \pi^+ \pi^- K^+}(\sqrt{s}, \sqrt{s_{2\text{th}}}),$$

$$\sigma_{pn \rightarrow \Lambda K^+ X}(\sqrt{s}, \sqrt{s_{\text{th}}}, \sqrt{s_{1\text{th}}}, \sqrt{s_{2\text{th}}}) = \frac{1}{2}\sigma_{pp \rightarrow \Lambda p K^+}(\sqrt{s}, \sqrt{s_{\text{th}}}) + \sigma_{pp \rightarrow \Lambda p \pi^+ K^0}(\sqrt{s}, \sqrt{s_{1\text{th}}}) \quad (31)$$

$$+ \frac{3}{2}\sigma_{pp \rightarrow \Lambda p \pi^+ \pi^- K^+}(\sqrt{s}, \sqrt{s_{2\text{th}}}) = \frac{1}{2}\sigma_{pp \rightarrow \Lambda K^+ X}(\sqrt{s}, \sqrt{s_{\text{th}}}, \sqrt{s_{1\text{th}}}, \sqrt{s_{2\text{th}}}).$$

Specify now the cross sections  $\sigma_{pN}^{\text{in}}$ ,  $\sigma_{K^+N}^{\text{tot}}$  and  $\sigma_{\Lambda N}^{\text{tot}}$ , entering into the formula (25). We adopt  $\sigma_{pN}^{\text{in}} = 30$  mb for the considered incident proton energy and  $\sigma_{K^+N}^{\text{tot}} = 12$  mb for all kaon momenta involved in our calculations [58]. Due to the isospin symmetry, the total cross sections  $\sigma_{\Lambda p}^{\text{tot}}$  and  $\sigma_{\Lambda n}^{\text{tot}}$  of the free  $\Lambda p$  and  $\Lambda n$  interactions are the same and we denoted their as  $\sigma_{\Lambda N}^{\text{tot}}$ . At  $\Lambda$  momenta of a few GeV/c of interest the cross section  $\sigma_{\Lambda p}^{\text{tot}}$  is entirely exhausted, as showed our calculations, by the total cross sections  $\sigma_{\Lambda p \rightarrow \Lambda p}$  and  $\sigma_{\Lambda p \rightarrow \Sigma^0 p}$ ,  $\sigma_{\Lambda p \rightarrow \Sigma^+ n}$  of elastic  $\Lambda p \rightarrow \Lambda p$  and inelastic  $\Lambda p \rightarrow \Sigma^0 p$ ,  $\Lambda p \rightarrow \Sigma^+ n$  processes. The isospin considerations show that  $\sigma_{\Lambda p \rightarrow \Sigma^+ n} = 2\sigma_{\Lambda p \rightarrow \Sigma^0 p}$ . With these, we have:

$$\sigma_{\Lambda N}^{\text{tot}} = \sigma_{\Lambda p}^{\text{tot}} = \sigma_{\Lambda p \rightarrow \Lambda p} + 3\sigma_{\Lambda p \rightarrow \Sigma^0 p}. \quad (32)$$

For the free lambda-proton total cross sections  $\sigma_{\Lambda p \rightarrow \Lambda p}$  and  $\sigma_{\Lambda p \rightarrow \Sigma^0 p}$  as functions of the laboratory  $\Lambda$  momentum  $p_\Lambda$  we employ the following parametrizations suggested in [60]:

$$\sigma_{\Lambda p \rightarrow \Lambda p}(p_\Lambda) = (39.66 - 100.45x + 92.44x^2 - 21.40x^3)/p_\Lambda \text{ [mb]}, \quad (33)$$

$$\sigma_{\Lambda p \rightarrow \Sigma^0 p}(p_\Lambda) = (31.10 - 30.94x + 8.16x^2)(p_\Sigma^{\text{cm}}/p_\Lambda^{\text{cm}}) \text{ [mb]}, \quad (34)$$

where  $x = \text{Min}(2.1 \text{ GeV/c}, p_\Lambda)$ ,  $p_\Lambda^{\text{cm}}$  and  $p_\Sigma^{\text{cm}}$  are the corresponding cm momenta. In formulas (33) and (34) the momenta are expressed in GeV/c. For the in-medium  $\Lambda p$  total cross section we use the relations (32)–(34), in which cm momenta  $p_\Lambda^{\text{cm}}$  and  $p_\Sigma^{\text{cm}}$  are defined as follows:

$$p_\Lambda^{\text{cm}} = \frac{1}{2\sqrt{s_\Lambda}}\lambda[s_\Lambda, m_N^2, (\langle m_\Lambda^* \rangle)^2], \quad p_\Sigma^{\text{cm}} = \frac{1}{2\sqrt{s_\Sigma}}\lambda[s_\Sigma, m_N^2, m_{\Sigma^0}^2], \quad s_\Sigma = s_\Lambda = (E'_\Lambda + m_N)^2 - p_\Lambda'^2, \quad (35)$$

where  $m_{\Sigma^0}$  is the  $\Sigma^0$  hyperon free space mass <sup>8)</sup> and the quantity  $\lambda(x, y, z)$  is defined in [57] by the equation (15).

Let us modify now the expression (24), describing the respective differential cross section for  $\Lambda K^+$  production in  $pA$  collisions from primary processes (1)–(6), to that corresponding to the kinematical conditions of the ANKE experiment. In this experiment, the differential cross section for production of  $\Lambda$  hyperons in the polar angular range of  $0^\circ \leq \theta_\Lambda \leq 6^\circ$  in lab system in the interaction of protons

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<sup>8)</sup>Following the predictions of the chiral effective field theory approach [20, 61],  $\text{SU}_6$  quark model [52] for the fate of hyperons in nuclear matter and phenomenological information inferred from hypernuclear data [1, 62] that the  $\Sigma$  hyperon experiences only a moderately repulsive nuclear potential of order 10–40 MeV at central nuclear densities and finite momenta as well as a weakly attractive one at the surface of the nucleus, we assume that the mass of  $\Sigma^0$  hyperon is not changed in the nuclear medium.

of energy of 2.83 GeV with the C, Cu, Ag, and Au target nuclei in coincidence with  $K^+$  mesons, which were required to have vacuum momenta in the interval of  $0.2 \text{ GeV}/c \leq p_{K^+} \leq 0.6 \text{ GeV}/c$  and to be in the polar angular domain of  $0^\circ \leq \theta_{K^+} \leq 12^\circ$ , was measured as a function of their vacuum momentum. Performing the respective integration of the full differential cross section (24) over ANKE acceptance window, we can represent this differential cross section in the following form:

$$\begin{aligned} & \left\langle \frac{d\sigma_{pA \rightarrow \Lambda X}^{(\text{prim})}(\mathbf{p}_0, p_\Lambda)}{dp_\Lambda d\Omega_\Lambda} \right\rangle_{\Delta\Omega_\Lambda \Delta\mathbf{p}_{K^+}} = \frac{1}{(2\pi)(1 - \cos 6^\circ)} \\ & \times \int_{0.2 \text{ GeV}/c}^{0.6 \text{ GeV}/c} dp_{K^+} \int_{\cos 12^\circ}^1 d\cos \theta_{K^+} \int_{\cos 6^\circ}^1 d\cos \theta_\Lambda \int_0^{2\pi} d\phi_{K^+} \int_0^{2\pi} d\phi_\Lambda \\ & \times \frac{d\sigma_{pA \rightarrow \Lambda K^+ X}^{(\text{prim})}(\mathbf{p}_0, \mathbf{p}_\Lambda, \mathbf{p}_{K^+})}{d\mathbf{p}_\Lambda d\mathbf{p}_{K^+}} p_\Lambda^2 p_{K^+}^2, \end{aligned} \quad (36)$$

where

$$\Delta\Omega_\Lambda = 2\pi(1 - \cos 6^\circ), \quad \Delta\mathbf{p}_{K^+} : 0.2 \text{ GeV}/c \leq p_{K^+} \leq 0.6 \text{ GeV}/c, \quad 0^\circ \leq \theta_{K^+} \leq 12^\circ. \quad (37)$$

Here,  $\phi_{K^+}$  and  $\phi_\Lambda$  are the azimuthal angles of kaon and  $\Lambda$  hyperon momenta  $\mathbf{p}_{K^+}$  and  $\mathbf{p}_\Lambda$  in lab system.

## 2.2. Two-step $\Lambda K^+$ production mechanisms

At incident energy of our interest the following two-step processes with pions in an intermediate states contribute mainly (see below) to the  $\Lambda K^+$  production in  $pA$  reactions <sup>9)</sup>:

$$p + N \rightarrow \pi^+, \pi^0, \pi^- + X; \quad (38)$$

$$\begin{aligned} \pi^+ + n &\rightarrow \Lambda + K^+, \\ \pi^0 + p &\rightarrow \Lambda + K^+; \end{aligned} \quad (39)$$

$$\begin{aligned} \pi^+ + p &\rightarrow \Lambda + \pi^+ + K^+, \\ \pi^0 + p &\rightarrow \Lambda + \pi^0 + K^+, \\ \pi^- + p &\rightarrow \Lambda + \pi^- + K^+, \\ \pi^+ + n &\rightarrow \Lambda + \pi^0 + K^+, \\ \pi^0 + n &\rightarrow \Lambda + \pi^- + K^+. \end{aligned} \quad (40)$$

Adopting the results given in [58], the differential  $\Lambda K^+$  production cross section for  $pA$  collisions at small laboratory angles from the secondary channels (39) and (40) can be represented as follows:

$$\begin{aligned} & \frac{d\sigma_{pA \rightarrow \Lambda K^+ X}^{(\text{sec}), (\pi)}(\mathbf{p}_0, \mathbf{p}_\Lambda, \mathbf{p}_{K^+})}{d\mathbf{p}_\Lambda d\mathbf{p}_{K^+}} = \frac{I_V^{\text{sec}}[A]}{I_V'[A]} \sum_{\pi'=\pi^+, \pi^0, \pi^-} \int_{4\pi} d\Omega_\pi \int_{p_\pi^{\text{abs}}}^{p_\pi^{\text{lim}}(\vartheta_\pi)} p_\pi^2 dp_\pi \frac{d\sigma_{pA \rightarrow \pi' X}^{(\text{prim})}(\mathbf{p}_0)}{d\mathbf{p}_\pi} \\ & \times \left[ \frac{Z}{A} \left\langle \frac{d\sigma_{\pi' p \rightarrow \Lambda K^+ X}(\mathbf{p}_\pi, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+})}{d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}} \right\rangle_A + \frac{N}{A} \left\langle \frac{d\sigma_{\pi' n \rightarrow \Lambda K^+ X}(\mathbf{p}_\pi, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+})}{d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}} \right\rangle_A \right] \frac{d\mathbf{p}'_\Lambda}{d\mathbf{p}_\Lambda} \frac{d\mathbf{p}'_{K^+}}{d\mathbf{p}_{K^+}}, \end{aligned} \quad (41)$$

<sup>9)</sup>Remind that the free threshold energies (or momenta), e.g., for the processes  $\pi^+ n \rightarrow \Lambda K^+$  and  $\pi^+ n \rightarrow \Lambda \pi^0 K^+$ , respectively, are 0.76 (0.89) and 1.0 GeV (1.13 GeV/c).

where

$$\begin{aligned}
I_V^{\text{sec}}[A] = & 2\pi A^2 \int_0^R r_\perp dr_\perp \int_{-\sqrt{R^2-r_\perp^2}}^{\sqrt{R^2-r_\perp^2}} dz \rho(\sqrt{r_\perp^2+z^2}) \int_0^{\sqrt{R^2-r_\perp^2}-z} dl \rho(\sqrt{r_\perp^2+(z+l)^2}) \\
& \times \exp \left[ -\sigma_{pN}^{\text{in}} A \int_{-\sqrt{R^2-r_\perp^2}}^z \rho(\sqrt{r_\perp^2+x^2}) dx - \sigma_{\pi'N}^{\text{tot}} A \int_z^{z+l} \rho(\sqrt{r_\perp^2+x^2}) dx \right] \\
& \times \exp \left[ -\sigma_{K^+N}^{\text{tot}} A \int_{z+l}^{\sqrt{R^2-r_\perp^2}} \rho(\sqrt{r_\perp^2+x^2}) dx - \sigma_{\Lambda N}^{\text{tot}}(p'_\Lambda) A \int_{z+l}^{\sqrt{R^2-r_\perp^2}} \rho(\sqrt{r_\perp^2+x^2}) dx \right]
\end{aligned} \tag{42}$$

and

$$\begin{aligned}
\left\langle \frac{d\sigma_{\pi'N \rightarrow \Lambda K^+ X}(\mathbf{p}_\pi, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+})}{d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}} \right\rangle_A = & \int \int P_A(\mathbf{p}_t, E) d\mathbf{p}_t dE \\
& \times \left\{ \frac{d\sigma_{\pi'N \rightarrow \Lambda K^+ X}[\sqrt{s_1}, < m_{K^+}^* >, < m_\Lambda^* >, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+}]}{d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}} \right\}.
\end{aligned} \tag{43}$$

Here,  $d\sigma_{\pi'N \rightarrow \Lambda K^+ X}[\sqrt{s_1}, < m_{K^+}^* >, < m_\Lambda^* >, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+}]/d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}$  are the "in-medium" differential cross sections for the  $\Lambda$  and  $K^+$  creation with effective masses  $< m_\Lambda^* >$  and  $< m_{K^+}^* >$  and with the in-medium momenta  $\mathbf{p}'_\Lambda$  and  $\mathbf{p}'_{K^+}$ , respectively, in reactions (39) and (40) at the  $\pi'N$  centre-of-mass energy  $\sqrt{s_1}$ . This quantity and the other ones, appearing in the equations (41) and (42), are defined in [44, 57, 59] as well as by the formulas (27), (32)–(35). Within the representation (39), (40), these cross sections can be written in the following forms:

$$\frac{d\sigma_{\pi^+ p \rightarrow \Lambda K^+ X}[\sqrt{s_1}, < m_{K^+}^* >, < m_\Lambda^* >, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+}]}{d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}} = \frac{d\sigma_{\pi^+ p \rightarrow \Lambda \pi^+ K^+}[\sqrt{s_1}, < m_{K^+}^* >, < m_\Lambda^* >, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+}]}{d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}}, \tag{44}$$

$$\begin{aligned}
\frac{d\sigma_{\pi^0 p \rightarrow \Lambda K^+ X}[\sqrt{s_1}, < m_{K^+}^* >, < m_\Lambda^* >, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+}]}{d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}} = & \frac{d\sigma_{\pi^0 p \rightarrow \Lambda K^+}[\sqrt{s_1}, < m_{K^+}^* >, < m_\Lambda^* >, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+}]}{d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}} \\
& + \frac{d\sigma_{\pi^0 p \rightarrow \Lambda \pi^0 K^+}[\sqrt{s_1}, < m_{K^+}^* >, < m_\Lambda^* >, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+}]}{d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}},
\end{aligned} \tag{45}$$

$$\frac{d\sigma_{\pi^- p \rightarrow \Lambda K^+ X}[\sqrt{s_1}, < m_{K^+}^* >, < m_\Lambda^* >, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+}]}{d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}} = \frac{d\sigma_{\pi^- p \rightarrow \Lambda \pi^- K^+}[\sqrt{s_1}, < m_{K^+}^* >, < m_\Lambda^* >, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+}]}{d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}}, \tag{46}$$

$$\begin{aligned}
\frac{d\sigma_{\pi^+ n \rightarrow \Lambda K^+ X}[\sqrt{s_1}, < m_{K^+}^* >, < m_\Lambda^* >, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+}]}{d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}} = & \frac{d\sigma_{\pi^+ n \rightarrow \Lambda K^+}[\sqrt{s_1}, < m_{K^+}^* >, < m_\Lambda^* >, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+}]}{d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}} \\
& + \frac{d\sigma_{\pi^+ n \rightarrow \Lambda \pi^0 K^+}[\sqrt{s_1}, < m_{K^+}^* >, < m_\Lambda^* >, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+}]}{d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}},
\end{aligned} \tag{47}$$

$$\frac{d\sigma_{\pi^0 n \rightarrow \Lambda K^+ X}[\sqrt{s_1}, < m_{K^+}^* >, < m_\Lambda^* >, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+}]}{d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}} = \frac{d\sigma_{\pi^0 n \rightarrow \Lambda \pi^- K^+}[\sqrt{s_1}, < m_{K^+}^* >, < m_\Lambda^* >, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+}]}{d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}}, \tag{48}$$

$$\frac{d\sigma_{\pi^- n \rightarrow \Lambda K^+ X}[\sqrt{s_1}, < m_{K^+}^* >, < m_\Lambda^* >, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+}]}{d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}} = 0, \tag{49}$$

where  $d\sigma_{\pi'N \rightarrow \Lambda K^+}[\sqrt{s_1}, < m_{K^+}^* >, < m_\Lambda^* >, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+}]/d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}$  and  $d\sigma_{\pi'N \rightarrow \Lambda \pi K^+}[\sqrt{s_1}, < m_{K^+}^* >, < m_\Lambda^* >, \mathbf{p}'_\Lambda, \mathbf{p}'_{K^+}]/d\mathbf{p}'_\Lambda d\mathbf{p}'_{K^+}$  are the "in-medium" differential cross sections of reaction channels (39)

and (40), correspondingly. Taking into account the two-body kinematics of elementary processes (39) as well as the isospin symmetry, we get the following expressions for the former ones:

$$\frac{d\sigma_{\pi^+n\rightarrow\Lambda K^+}[\sqrt{s_1}, < m_{K^+}^* >, < m_{\Lambda}^* >, \mathbf{p}'_{\Lambda}, \mathbf{p}'_{K^+}]}{d\mathbf{p}'_{\Lambda} d\mathbf{p}'_{K^+}} = \frac{\pi}{I_2(s_1, < m_{K^+}^* >, < m_{\Lambda}^* >) E'_{\Lambda}} \quad (50)$$

$$\begin{aligned} & \times \frac{d\sigma_{\pi^+n\rightarrow\Lambda K^+}(\sqrt{s_1}, < m_{K^+}^* >, < m_{\Lambda}^* >, \theta_{\Lambda}^*)}{d\Omega_{\Lambda}^*} \\ & \times \frac{1}{(\omega_0 + E_t)} \delta \left[ \omega_0 + E_t - \sqrt{(< m_{K^+}^* >)^2 + \mathbf{p}_{K^+}^{\prime 2}} \right] \delta(\mathbf{Q}_0 + \mathbf{p}_t - \mathbf{p}'_{K^+}), \\ \frac{d\sigma_{\pi^0 p \rightarrow \Lambda K^+}[\sqrt{s_1}, < m_{K^+}^* >, < m_{\Lambda}^* >, \mathbf{p}'_{\Lambda}, \mathbf{p}'_{K^+}]}{d\mathbf{p}'_{\Lambda} d\mathbf{p}'_{K^+}} &= \frac{1}{2} \frac{d\sigma_{\pi^+n\rightarrow\Lambda K^+}[\sqrt{s_1}, < m_{K^+}^* >, < m_{\Lambda}^* >, \mathbf{p}'_{\Lambda}, \mathbf{p}'_{K^+}]}{d\mathbf{p}'_{\Lambda} d\mathbf{p}'_{K^+}}. \end{aligned} \quad (51)$$

Here,

$$\omega_0 = E_{\pi} - E'_{\Lambda}, \quad \mathbf{Q}_0 = \mathbf{p}_{\pi} - \mathbf{p}'_{\Lambda}, \quad (52)$$

$\mathbf{p}_{\pi}$  and  $E_{\pi}$  are the momentum and total energy of an intermediate pion (which is assumed to be on-shell),  $I_2$  is the two-body phase space defined in [58] by the formula (17). In equation (50),  $d\sigma_{\pi^+n\rightarrow\Lambda K^+}(\sqrt{s_1}, < m_{K^+}^* >, < m_{\Lambda}^* >, \theta_{\Lambda}^*)/d\Omega_{\Lambda}^*$  is the  $\Lambda$  "in-medium" differential cross section in the  $\pi^+n$  center-of-mass system. As an earlier, we assume that this cross section is equivalent to the vacuum one  $d\sigma_{\pi^+n\rightarrow\Lambda K^+}(\sqrt{s_1}, m_{K^+}, m_{\Lambda}, \theta_{\Lambda}^*)/d\Omega_{\Lambda}^*$  in which the free kaon and  $\Lambda$  hyperon masses  $m_{K^+}$  and  $m_{\Lambda}$  are replaced by the in-medium masses  $< m_{K^+}^* >$  and  $< m_{\Lambda}^* >$ . According to [63, 64], we choose the free  $\Lambda$  angular distribution in the following form:

$$\frac{d\sigma_{\pi^+n\rightarrow\Lambda K^+}(\sqrt{s_1}, m_{K^+}, m_{\Lambda}, \theta_{\Lambda}^*)}{d\Omega_{\Lambda}^*} = [1 - A_1(\sqrt{s_1}, \sqrt{\tilde{s}_{\text{th}}}) \cos \theta_{\Lambda}^*] \frac{\sigma_{\pi^+n\rightarrow\Lambda K^+}(\sqrt{s_1}, \sqrt{\tilde{s}_{\text{th}}})}{4\pi}, \quad (53)$$

$$A_1(\sqrt{s_1}, \sqrt{\tilde{s}_{\text{th}}}) = \begin{cases} 5.26 \left( \frac{\sqrt{s_1} - \sqrt{\tilde{s}_{\text{th}}}}{\text{GeV}} \right) & \text{for } \sqrt{\tilde{s}_{\text{th}}} < \sqrt{s_1} \leq 1.8 \text{ GeV}, \\ 1 & \text{for } \sqrt{s_1} > 1.8 \text{ GeV}, \end{cases} \quad (54)$$

$$\sigma_{\pi^+n\rightarrow\Lambda K^+}(\sqrt{s_1}, \sqrt{\tilde{s}_{\text{th}}}) = \begin{cases} 10.0 \left( \frac{\sqrt{s_1} - \sqrt{\tilde{s}_{\text{th}}}}{\text{GeV}} \right) [\text{mb}] & \text{for } \sqrt{\tilde{s}_{\text{th}}} < \sqrt{s_1} \leq 1.7 \text{ GeV}, \\ 0.09 \left( \frac{\text{GeV}}{\sqrt{s_1} - 1.6 \text{ GeV}} \right) [\text{mb}] & \text{for } \sqrt{s_1} > 1.7 \text{ GeV}, \end{cases} \quad (55)$$

where  $\sqrt{\tilde{s}_{\text{th}}} = m_{\Lambda} + m_{K^+}$  is the free threshold energy. In our calculations of the  $\Lambda K^+$  pair production on nuclei the "in-medium" differential cross sections  $d\sigma_{\pi'N\rightarrow\Lambda\pi K^+}[\sqrt{s_1}, < m_{K^+}^* >, < m_{\Lambda}^* >, \mathbf{p}'_{\Lambda}, \mathbf{p}'_{K^+}]/d\mathbf{p}'_{\Lambda} d\mathbf{p}'_{K^+}$  of reaction channels (40) have been described according to the three-body phase space. Following (29), one has:

$$\begin{aligned} & \frac{d\sigma_{\pi'N\rightarrow\Lambda\pi K^+}[\sqrt{s_1}, < m_{K^+}^* >, < m_{\Lambda}^* >, \mathbf{p}'_{\Lambda}, \mathbf{p}'_{K^+}]}{d\mathbf{p}'_{\Lambda} d\mathbf{p}'_{K^+}} = \frac{1}{8E'_{\Lambda} E'_{K^+}} \\ & \times \frac{\sigma_{\pi'N\rightarrow\Lambda\pi K^+}(\sqrt{s_1}, \sqrt{\tilde{s}_{1\text{th}}^*})}{I_3(s_1, < m_{K^+}^* >, m_{\pi}, < m_{\Lambda}^* >)} \frac{1}{(\omega_1 + E_t)} \delta \left( \omega_1 + E_t - \sqrt{m_{\pi}^2 + (\mathbf{Q}_1 + \mathbf{p}_t)^2} \right), \end{aligned} \quad (56)$$

where

$$\omega_1 = E_{\pi} - E'_{\Lambda} - E'_{K^+}, \quad \mathbf{Q}_1 = \mathbf{p}_{\pi} - \mathbf{p}'_{\Lambda} - \mathbf{p}'_{K^+} \quad (57)$$

and  $\sqrt{\tilde{s}_{1\text{th}}^*} = < m_{\Lambda}^* > + m_{\pi} + < m_{K^+}^* >$  is the in-medium threshold energy. In equation (56),  $\sigma_{\pi'N\rightarrow\Lambda\pi K^+}(\sqrt{s_1}, \sqrt{\tilde{s}_{1\text{th}}^*})$  are the "in-medium" total cross sections for the  $\Lambda K^+$  pair production in reactions (40). As before, we assume that these cross sections are equivalent to the vacuum ones

$\sigma_{\pi'N \rightarrow \Lambda\pi K^+}(\sqrt{s_1}, \sqrt{\tilde{s}_{1\text{th}}})$  in which the free threshold energy  $\sqrt{\tilde{s}_{1\text{th}}} = m_\Lambda + m_\pi + m_{K^+}$  is replaced by the in-medium threshold  $\sqrt{\tilde{s}_{1\text{th}}^*}$ . In line with [35], for the free total cross sections  $\sigma_{\pi'N \rightarrow \Lambda\pi K^+}(\sqrt{s_1}, \sqrt{\tilde{s}_{1\text{th}}})$  we have adopted the following expression:

$$\begin{aligned} \sigma_{\pi^+p \rightarrow \Lambda\pi^+K^+}(\sqrt{s_1}, \sqrt{\tilde{s}_{1\text{th}}}) &\approx \sigma_{\pi^0p \rightarrow \Lambda\pi^0K^+}(\sqrt{s_1}, \sqrt{\tilde{s}_{1\text{th}}}) \approx \sigma_{\pi^-p \rightarrow \Lambda\pi^-K^+}(\sqrt{s_1}, \sqrt{\tilde{s}_{1\text{th}}}) \\ &\approx \sigma_{\pi^+n \rightarrow \Lambda\pi^0K^+}(\sqrt{s_1}, \sqrt{\tilde{s}_{1\text{th}}}) \approx \sigma_{\pi^0n \rightarrow \Lambda\pi^-K^+}(\sqrt{s_1}, \sqrt{\tilde{s}_{1\text{th}}}) \approx 24.0 \left(1 - \frac{\tilde{s}_{1\text{th}}}{s_1}\right)^{3.16} \left(\frac{\tilde{s}_{1\text{th}}}{s_1}\right)^{4.24} [\text{mb}]. \end{aligned} \quad (58)$$

It is worthwhile mentioning that, as follows from the equations (55), (58) and compilation [65], the  $\Lambda K^+$  production cross sections in secondary pion–nucleon processes (39) and (40) are substantially larger than those in the four-body reaction channels  $\pi'N \rightarrow \Lambda\pi\pi K^+$  at pion momenta  $\leq 2$  GeV/c, giving, as showed our estimate, the main contribution to the  $\Lambda K^+$  creation on nuclei for kinematics of interest. Therefore, we discard the latter in the present study.

In addition to the two-step processes with intermediate pions (38)–(40) we consider the following production/decay sequence, which may contribute to the  $\Lambda K^+$  on nuclei in the ANKE acceptance window at incident proton beam energy of interest:

$$p + p \rightarrow \Sigma^0 + p + K^+, \quad (59)$$

$$p + n \rightarrow \Sigma^0 + n + K^+, \quad (60)$$

$$\Sigma^0 \rightarrow \Lambda + \gamma. \quad (61)$$

Presently, there four sets of data are available for the total cross section  $\sigma_{pp \rightarrow \Sigma^0 p K^+}$  of reaction (59). Three of them have been recently taken by the COSY-11 [24, 25], COSY-TOF [29] and ANKE [30] Collaborations at proton energies  $\leq 2.4$  GeV, whereas the one was obtained a long time ago at beam energies  $\geq 2.85$  GeV [21]. The comparison of these data with the results of calculations by parametrization

$$\sigma_{pp \rightarrow \Sigma^0 p K^+}(\sqrt{s}, \sqrt{s_0}) = \begin{cases} \frac{A_{\Sigma^0}(s-s_0)^2}{4m_p^2 + B_{\Sigma^0}(s-s_0)^2} & \text{for } \sqrt{s} - \sqrt{s_0} > 0.225 \text{ GeV}, \\ C_{\Sigma^0}(\sqrt{s} - \sqrt{s_0})^2 & \text{for } 0 < \sqrt{s} - \sqrt{s_0} \leq 0.225 \text{ GeV}, \end{cases} \quad (62)$$

(solid line) is shown in figure 4. In (62),  $\sqrt{s_0} = m_{\Sigma^0} + m_p + m_{K^+}$  is the threshold energy and the constants  $A_{\Sigma^0}$ ,  $B_{\Sigma^0}$  and  $C_{\Sigma^0}$  are given as  $26.0 \mu\text{b}/\text{GeV}^2$ ,  $1/\text{GeV}^2$  and  $154.5 \mu\text{b}/\text{GeV}^2$ , respectively. The "low" excess energy part of formula (62) was taken from the work [29]. It can be seen that the parametrization (62) fits well the full available set of data for the  $pp \rightarrow \Sigma^0 p K^+$  process <sup>10)</sup>. Direct data on the total cross section  $\sigma_{pn \rightarrow \Sigma^0 n K^+}$  of reaction (60) do not exist currently. There is [21] only three data points for the total cross section  $\sigma_{pn \rightarrow \Sigma^0 p K^0}$  of the channel  $pn \rightarrow \Sigma^0 p K^0$  at 5.135, 6.124 and 16.088 GeV initial energies, which due to the isospin symmetry is equal to the former one. Comparing them with the data that are available [21] for the  $pp \rightarrow \Sigma^0 p K^+$  process at similar energies (at energies of 5.135 and 6.045 GeV), one can readily get that at these energies the ratio of the  $pn$  and  $pp$  total cross sections  $\sigma_{pn \rightarrow \Sigma^0 p K^0}/\sigma_{pp \rightarrow \Sigma^0 p K^+}$  varies approximately from 0.5 to 1.5. Therefore, it is natural to take for this ratio here an average value of one, which means that:

$$\sigma_{pn \rightarrow \Sigma^0 p K^0}(\sqrt{s}) = \sigma_{pn \rightarrow \Sigma^0 n K^+}(\sqrt{s}) \approx \sigma_{pp \rightarrow \Sigma^0 p K^+}(\sqrt{s}, \sqrt{s_0}) \quad (63)$$

<sup>10)</sup>It should be mentioned that this parametrization is also consistent with the cross section of  $(16.5 \pm 20\%) \mu\text{b}$  for channel  $pp \rightarrow \Sigma^0 p K^+$  at beam energy of 3.5 GeV ( $\sqrt{s} - \sqrt{s_0} = 0.555$  GeV) evaluated in [31] by dividing the measured cross section for  $pp \rightarrow \Lambda p K^+$  reaction at this energy by a factor of 2.2 and assuming an uncertainty of 20%.



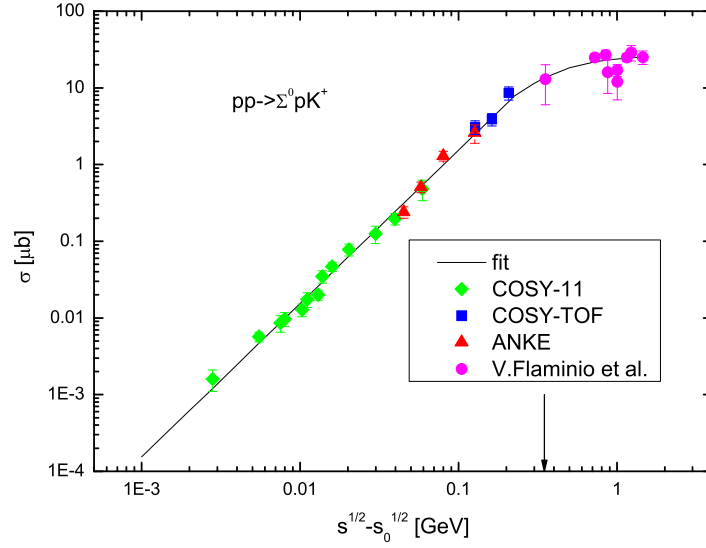


Figure 4: Total cross section for  $pp \rightarrow \Sigma^0 p K^+$  reaction as a function of an excess energy. The arrow indicates the excess energy, which corresponds to the proton kinetic energy of 2.83 GeV. For further notation see the text.

at the considered high incident proton kinetic energies. Due to the lack of the respective data at lower beam energies, we will adopt the relation (63) also at all collision energies  $\sqrt{s}$ , accessible in the calculation of  $\Lambda K^+$  production in  $pA$  interactions from the production/decay sequence (59)–(61) at initial energy of 2.83 GeV of interest. At this energy the ratio  $R_{\Sigma^0/\Lambda}$  of the total cross sections  $\sigma_{pp \rightarrow \Sigma^0 p K^+}$  and  $\sigma_{pp \rightarrow \Lambda p K^+}$ , shown in figures 4 and 1, is about 1/4. Hence, the production/decay chain (59)–(61) may indeed contribute to the  $(p, \Lambda K^+)$  reaction on nuclei for initial energy of our interest. It should be noted that the total cross section of the subprocess  $pp \rightarrow \Sigma^0 p \pi^0 K^+$  with additional pion in final state, assuming that it goes completely through the reaction  $pp \rightarrow \Lambda(1405) p K^+$ , is  $(1.5 \pm 0.7) \mu\text{b}$  at beam energy of 2.83 GeV [66]. This cross section is substantially lower than that for the  $pp \rightarrow \Sigma^0 p K^+$  process, shown in figure 4, at latter energy. Therefore, we will neglect the contribution from the production/decay chain  $pN \rightarrow \Sigma^0 N \pi^0 K^+$ ,  $\Sigma^0 \rightarrow \Lambda \gamma$  in our calculations of the  $\Lambda K^+$  yield in  $pA$  collisions at 2.83 GeV incident energy.

Taking into account the facts that the produced in elementary processes (59), (60)  $\Sigma^0$  hyperons and  $K^+$  mesons are mainly emitted in forward direction and the most of  $\Sigma^0$ 's decay into  $\Lambda$  and  $\gamma$  essentially outside the target nuclei of interest as well as neglecting the change of the  $\Sigma^0$  mass in the nuclear medium but accounting for the in-medium modifications of the masses of other final hadrons (kaon and nucleon) participating in these processes on the same footing as that employed in calculating the  $\Lambda K^+$  production cross section (24) from the primary proton-induced reaction channels (1)–(6) and using the results given in [67, 68], we get the following expression for the  $\Lambda K^+$  creation cross section for  $pA$  interactions from this chain:

$$\begin{aligned}
 & \frac{d\sigma_{pA \rightarrow \Lambda K^+ X}^{(\text{sec}), (\Sigma^0)}(\mathbf{p}_0, \mathbf{p}_\Lambda, \mathbf{p}_{K^+})}{d\mathbf{p}_\Lambda d\mathbf{p}_{K^+}} = I_{K^+ \Sigma^0}[A] \int d\mathbf{p}_{\Sigma^0} \\
 & \times \left[ \frac{Z}{A} \left\langle \frac{d\sigma_{pp \rightarrow \Sigma^0 p K^+}(\mathbf{p}'_0, \mathbf{p}_{\Sigma^0}, \mathbf{p}'_{K^+})}{d\mathbf{p}_{\Sigma^0} d\mathbf{p}'_{K^+}} \right\rangle_A + \frac{N}{A} \left\langle \frac{d\sigma_{pn \rightarrow \Sigma^0 n K^+}(\mathbf{p}'_0, \mathbf{p}_{\Sigma^0}, \mathbf{p}'_{K^+})}{d\mathbf{p}_{\Sigma^0} d\mathbf{p}'_{K^+}} \right\rangle_A \right] \frac{d\mathbf{p}'_{K^+}}{d\mathbf{p}_{K^+}} \\
 & \times \frac{BR(\Sigma^0 \rightarrow \Lambda \gamma) \theta(E_{\Sigma^0} - E_\Lambda)}{4I_2(m_{\Sigma^0}^2, m_\Lambda^2, 0) E_\Lambda (E_{\Sigma^0} - E_\Lambda)} \delta \left( E_{\Sigma^0} - E_\Lambda - \sqrt{(\mathbf{p}_{\Sigma^0} - \mathbf{p}_\Lambda)^2} \right),
 \end{aligned} \tag{64}$$

where  $\mathbf{p}_{\Sigma^0}$  and  $E_{\Sigma^0}$  are the vacuum momentum and total energy of a  $\Sigma^0$  hyperon ( $E_{\Sigma^0} = \sqrt{m_{\Sigma^0}^2 + \mathbf{p}_{\Sigma^0}^2}$ ),  $\theta(x)$  is the standard step function and  $BR(\Sigma^0 \rightarrow \Lambda\gamma) = 1$ . The averaged differential cross sections  $\langle d\sigma_{pN \rightarrow \Sigma^0 NK^+}(\mathbf{p}'_0, \mathbf{p}_{\Sigma^0}, \mathbf{p}'_{K^+})/d\mathbf{p}_{\Sigma^0}d\mathbf{p}'_{K^+} \rangle_A$ , entering into the equation (64), are defined by the formulas (26), (28), (29) in which one has to make the following substitutions:  $\mathbf{p}'_{\Lambda} \rightarrow \mathbf{p}_{\Sigma^0}$ ,  $E'_{\Lambda} \rightarrow E_{\Sigma^0}$ ,  $< m_{\Lambda}^* > \rightarrow m_{\Sigma^0}$  and  $\sigma_{pN \rightarrow \Lambda K^+ X}(\sqrt{s}, \sqrt{s_{1th}^*}, \sqrt{s_{2th}^*}, \sqrt{s_{2th}^*}) \rightarrow \sigma_{pN \rightarrow \Sigma^0 NK^+}(\sqrt{s}, \sqrt{s_0^*})$ , where  $\sqrt{s_0^*} = m_{\Sigma^0} + < m_p^* > + < m_{K^+}^* >$ . The quantity  $I_{K^+\Sigma^0}[A]$  in (64) is defined above by equation (25) in which one has to replace  $\sigma_{\Lambda N}^{\text{tot}}$  on inelastic cross section  $\sigma_{\Sigma^0 N}^{\text{in}}$  of the  $\Sigma^0 N$  interaction<sup>11)</sup>. Due to isospin symmetry, this cross section is the same as the inelastic cross sections  $\sigma_{\Sigma^0 p}^{\text{in}}$  and  $\sigma_{\Sigma^0 n}^{\text{in}}$  of the  $\Sigma^0 p$  and  $\Sigma^0 n$  interactions. At  $\Sigma^0$  momenta of interest the cross section  $\sigma_{\Sigma^0 p}^{\text{in}}$  is exhausted by the total cross sections  $\sigma_{\Sigma^0 p \rightarrow \Lambda p}$  and  $\sigma_{\Sigma^0 p \rightarrow \Sigma^+ n}$  of inelastic  $\Sigma^0 p \rightarrow \Lambda p$  and  $\Sigma^0 p \rightarrow \Sigma^+ n$  processes:

$$\sigma_{\Sigma^0 N}^{\text{in}} = \sigma_{\Sigma^0 p}^{\text{in}} = \sigma_{\Sigma^0 p \rightarrow \Lambda p} + \sigma_{\Sigma^0 p \rightarrow \Sigma^+ n}. \quad (65)$$

The first cross section in (65) is obtained by detailed balance [60]:

$$\sigma_{\Sigma^0 p \rightarrow \Lambda p} = \left( \frac{p_{\Lambda}^{\text{cm}}}{p_{\Sigma}^{\text{cm}}} \right)^2 \sigma_{\Lambda p \rightarrow \Sigma^0 p}(p'_{\Lambda}), \quad (66)$$

where the quantities  $\sigma_{\Lambda p \rightarrow \Sigma^0 p}$ ,  $p_{\Lambda}^{\text{cm}}$  and  $p_{\Sigma}^{\text{cm}}$  are defined above by the formulas (34), (35), in which one has to put:

$$s_{\Sigma} = (E_{\Sigma^0} + m_N)^2 - p_{\Sigma^0}^2, \quad p'_{\Lambda} = \sqrt{E_{\Lambda}^2 - (< m_{\Lambda}^* >)^2}, \quad E'_{\Lambda} = [s_{\Sigma} - m_N^2 - (< m_{\Lambda}^* >)^2]/(2m_N). \quad (67)$$

For the second cross section in (65) we adopt the following parametrization suggested in [60]:

$$\sigma_{\Sigma^0 p \rightarrow \Sigma^+ n}(p_{\Sigma^0}) = 22.4/p_{\Sigma^0} - 1.08 \text{ [mb]}, \quad (68)$$

where the  $\Sigma^0$  momentum  $p_{\Sigma^0}$  is measured in GeV/c.

The differential cross section for  $\Lambda$  hyperon production in  $pA$  collisions in coincidence with the  $K^+$  meson from the two-step processes (38)–(40) and (59)–(61), corresponding to the kinematical conditions of the ANKE experiment, can be analogously to (36) defined as:

$$\begin{aligned} & \left\langle \frac{d\sigma_{pA \rightarrow \Lambda X}^{(\text{sec})}(\mathbf{p}_0, p_{\Lambda})}{dp_{\Lambda} d\Omega_{\Lambda}} \right\rangle_{\Delta\Omega_{\Lambda} \Delta\mathbf{p}_{K^+}} = \frac{1}{(2\pi)(1 - \cos 6^\circ)} \\ & \times \int_{0.2 \text{ GeV/c}}^{0.6 \text{ GeV/c}} dp_{K^+} \int_{\cos 12^\circ}^1 d\cos \theta_{K^+} \int_{\cos 6^\circ}^1 d\cos \theta_{\Lambda} \int_0^{2\pi} d\phi_{K^+} \int_0^{2\pi} d\phi_{\Lambda} \\ & \times \left[ \frac{d\sigma_{pA \rightarrow \Lambda K^+ X}^{(\text{sec}), (\pi)}(\mathbf{p}_0, \mathbf{p}_{\Lambda}, \mathbf{p}_{K^+})}{d\mathbf{p}_{\Lambda} d\mathbf{p}_{K^+}} + \frac{d\sigma_{pA \rightarrow \Lambda K^+ X}^{(\text{sec}), (\Sigma^0)}(\mathbf{p}_0, \mathbf{p}_{\Lambda}, \mathbf{p}_{K^+})}{d\mathbf{p}_{\Lambda} d\mathbf{p}_{K^+}} \right] p_{\Lambda}^2 p_{K^+}^2. \end{aligned} \quad (69)$$

Discuss now the results of calculations within the approach outlined above.

### 3. Results

At first, we consider the differential  $\Lambda$  production cross sections in the ANKE acceptance window from the one-step, two-step and one- plus two-step creation mechanisms in  $pC$  and  $pAu$

<sup>11)</sup>Using this cross section in (25), we assume that the quasi-elastic  $\Sigma^0$  rescatterings on intranuclear nucleons do not lead to the loss of  $\Sigma^0$  hyperons, which may undergo subsequently the  $\Sigma^0 \rightarrow \Lambda\gamma$  decays.

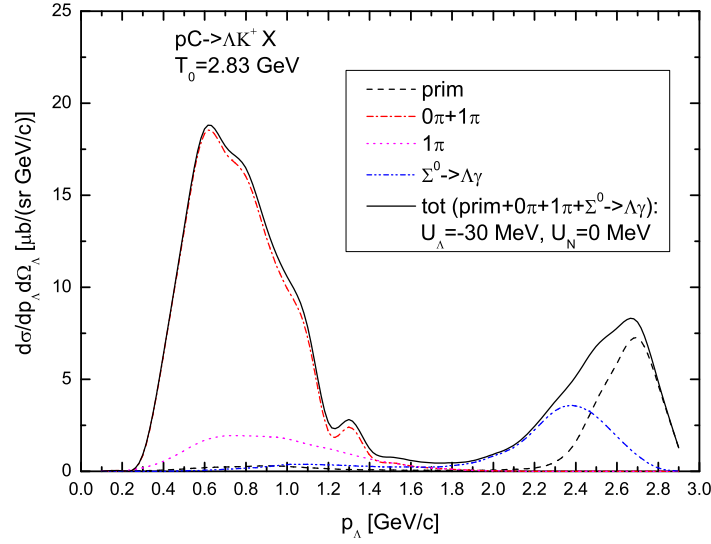


Figure 5: Differential cross section for the production of  $\Lambda$  hyperons in coincidence with the  $K^+$  mesons from primary (1)–(6) (dashed line), secondary (39), (40) (dotted-dashed line), secondary (40) (dotted line), secondary (61) (dot-dot-dashed line) and primary (1)–(6) plus secondary (39), (40), (61) (solid line) channels in the ANKE acceptance window as a function of lambda momentum in the interaction of protons of energy of 2.83 GeV with C target nucleus for  $\Lambda$  potential depth  $U_\Lambda = -30 \text{ MeV}$ .

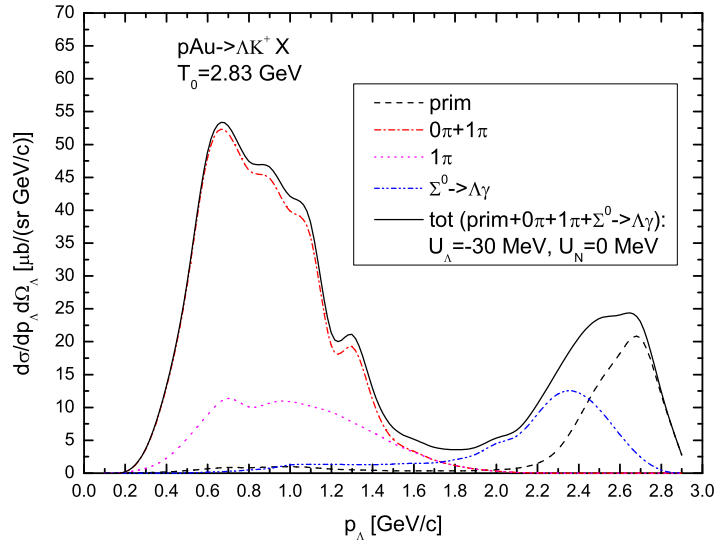


Figure 6: The same as in figure 5, but for the Au target nucleus.

reactions at 2.83 GeV beam energy, calculated on the basis of equations (36) and (69) in the scenario for the  $\Lambda$  potential depth  $U_\Lambda = -30 \text{ MeV}$ . These cross sections are presented in figures 5 and 6, correspondingly. One can see that the secondary  $\Lambda K^+$  production processes (39), (40) with a pion in an intermediate states are of importance compared to the primary ones (1)–(6) and to the

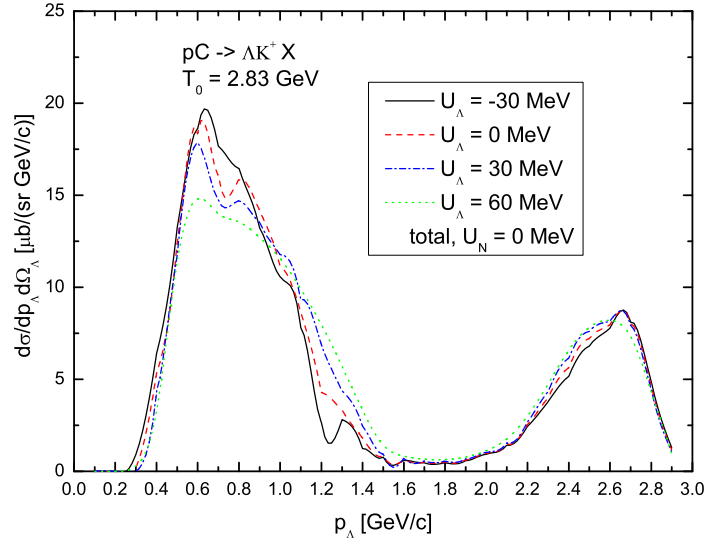


Figure 7: Differential cross section for the production of  $\Lambda$  hyperons in coincidence with the  $K^+$  mesons from primary plus secondary channels in the ANKE acceptance window as a function of lambda momentum in the interaction of protons of energy of 2.83 GeV with C target nucleus for effective scalar  $\Lambda$  potentials at saturation density  $U_\Lambda = -30 \text{ MeV}$  (solid line),  $U_\Lambda = 0 \text{ MeV}$  (dashed line),  $U_\Lambda = 30 \text{ MeV}$  (dotted-dashed line) and  $U_\Lambda = 60 \text{ MeV}$  (dotted line).

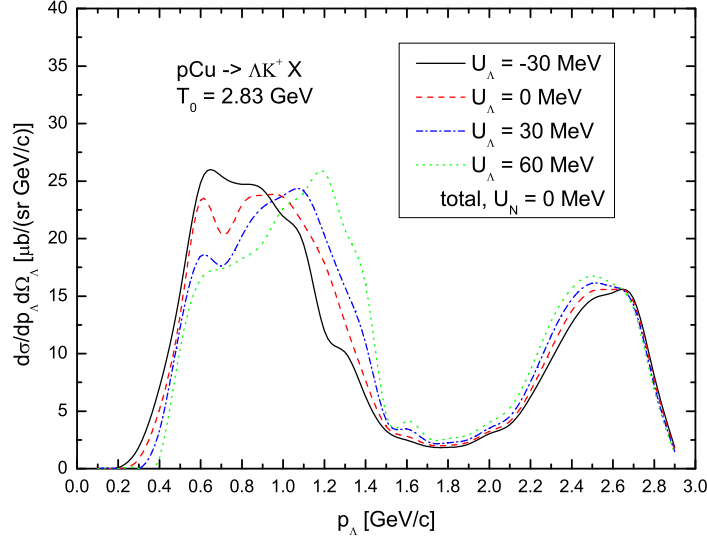


Figure 8: The same as in figure 7, but for the Cu target nucleus.

secondary process (61) associated with the production of  $\Lambda$ s via the vacuum decay of intermediate  $\Sigma^0$  hyperons in the chosen kinematics at laboratory lambda momenta  $\leq 1.8 \text{ GeV}/c$  for both target nuclei and the dominance here is substantially more pronounced for the  $\pi N \rightarrow \Lambda K^+$  channels. Whereas at higher  $\Lambda$  momenta around of 2.4 and 2.7  $\text{GeV}/c$  the two-step with intermediate  $\Sigma^0$  hyperons and one-step creation mechanisms are, respectively, dominant. Evidently, the dependence

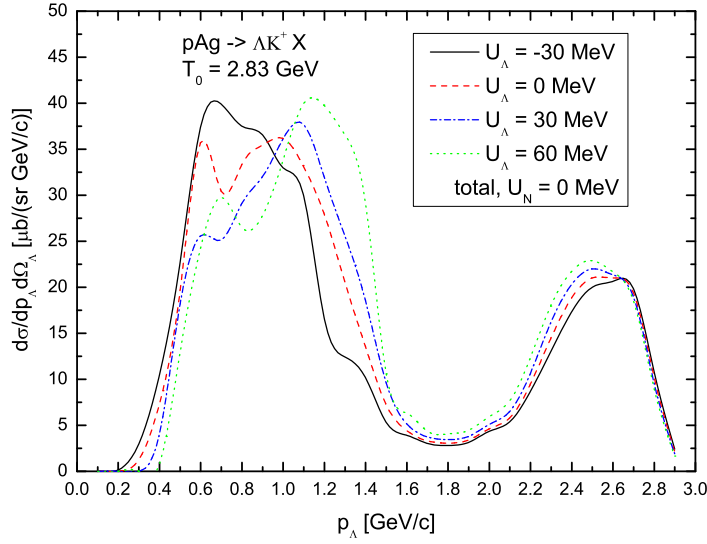


Figure 9: The same as in figure 7, but for the Ag target nucleus.

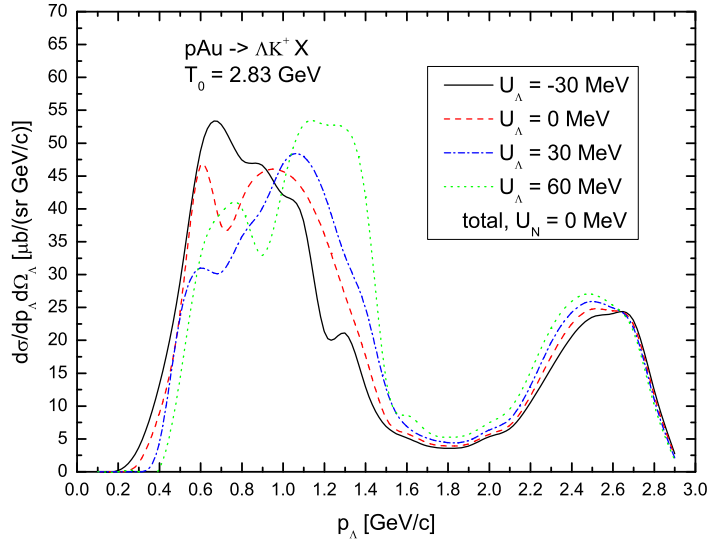


Figure 10: The same as in figure 7, but for the Au target nucleus.

of the considered coincident  $\Lambda$  spectrum on the  $\Lambda$  potential should exist mainly at low lambda momenta (cf. figures 7–10 given below). This means that the secondary pion–nucleon production processes have to be accounted for in the analysis of the data on  $\Lambda K^+$  pair creation in  $pA$  collisions obtained in the ANKE experiment with the aim of extracting the  $\Lambda$ –nuclear potential. It should be noted that the relative roles of the individual  $\Lambda K^+$  production channels for another considered options for this potential at saturation density and target nuclei are similar to those, illustrated in figures 5 and 6 for its depth  $U_\Lambda = -30$  MeV for C and Au targets.

In figures 7–10 we show the results of our calculations following equations (36) and (69) for the overall differential cross sections for the production of  $\Lambda$  hyperons in coincidence with the  $K^+$  mesons

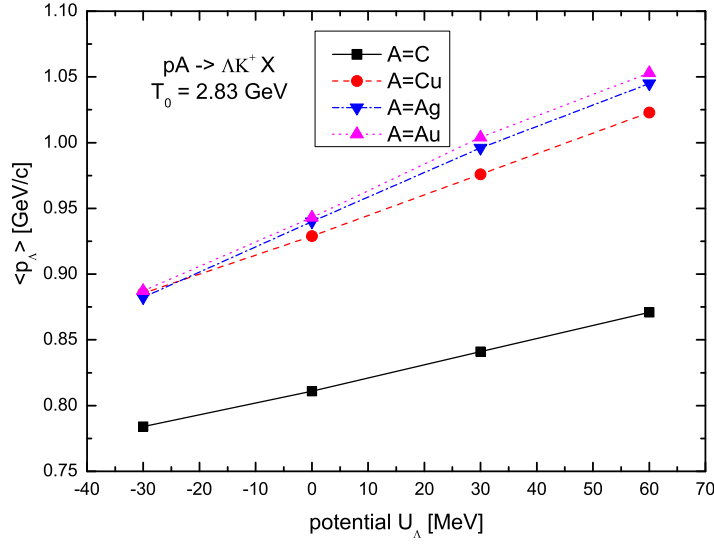


Figure 11: Average momenta of  $\Lambda$  hyperons in the low-momentum parts of their spectra, shown in figures 7–10, from  $pC$ ,  $pCu$ ,  $pAg$  and  $pAu$  interactions at an incident energy of 2.83 GeV as functions of the effective scalar  $\Lambda$  potential  $U_{\Lambda}$  at normal nuclear density. The lines are to guide the eyes.

on C, Cu, Ag and Au target nuclei in the kinematical conditions of the ANKE experiment, obtained for incident energy of 2.83 GeV by adopting in them four options for the effective scalar hyperon potential  $U_{\Lambda}$  at normal nuclear matter density indicated in the insets. It is indeed clearly seen that these cross sections are appreciably sensitive to the  $\Lambda$  potential at momenta less than 1.8 GeV/c for all considered target nuclei, namely: their strengths shift to higher momenta with increasing the  $\Lambda$  potential  $U_{\Lambda}$  up to 60 MeV. The sensitivity of the strength of the low-momentum part of  $\Lambda$  spectrum on the scalar lambda potential  $U_{\Lambda}$ , exhibited in figures (7)–(10), can be exploited to infer this potential from the direct comparison of the shapes of the calculated  $\Lambda$  hyperon differential distributions with those determined in the ANKE experiment. As an additional measure for the correlation between the above strength and the  $\Lambda$ –nuclear potential  $U_{\Lambda}$  we choose the average

momentum  $\langle p_{\Lambda} \rangle$ , defined as  $\langle p_{\Lambda} \rangle = \frac{\int_{0.1 \text{ GeV/c}}^{1.8 \text{ GeV/c}} p_{\Lambda} dp_{\Lambda} d\sigma / dp_{\Lambda} d\Omega_{\Lambda}}{\int_{0.1 \text{ GeV/c}}^{1.8 \text{ GeV/c}} dp_{\Lambda} d\sigma / dp_{\Lambda} d\Omega_{\Lambda}}$ , where

$d\sigma / dp_{\Lambda} d\Omega_{\Lambda}$  are the  $\Lambda$  differential cross sections presented in figures 7–10. This momentum as a function of potential  $U_{\Lambda}$  is plotted in figure 11. One can see that the carbon nucleus is not optimal for determining this potential. The highest sensitivity to it we have for heavy silver and gold target nuclei. Thus, for example, for gold nucleus the difference between the mean momenta  $\langle p_{\Lambda} \rangle$ , corresponding to the  $\Lambda$  potential at saturation density  $U_{\Lambda} = -30$  MeV and  $U_{\Lambda} = 60$  MeV, is 166 MeV/c. Whereas the same difference for carbon target nucleus is only 87 MeV/c<sup>12)</sup>. Therefore, a comparison above results with the experimentally determined average momentum in the low-momentum part of the  $\Lambda$  spectrum on heavy target nuclei under consideration will also allow one to

<sup>12)</sup>It is interesting to note that the analogous difference between the average momenta, corresponding to the high-momentum region of 1.8–2.9 GeV/c of the  $\Lambda$  spectrum on Au nucleus as well as to the same potentials  $U_{\Lambda} = -30$  MeV and  $U_{\Lambda} = 60$  MeV, amounts, as showed our calculations, to 42 MeV/c, what demonstrates very moderate sensitivity of the considered momentum distributions to the adopted  $\Lambda$  in-medium modification scenarios also at high momenta of interest.

deduce the effective scalar potential  $U_\Lambda$  in cold nuclear matter at this momentum <sup>13)</sup>. Knowing this potential and using the relations (19), (20), we can easily recover the single-particle potential  $V_{\Lambda\Lambda}^{\text{SEP}}$  at saturation density for in-medium momentum  $p'_\Lambda$ , corresponding to the experimentally determined average momentum  $\langle p_\Lambda \rangle$ . Such data point may help to discriminate between the existing models of the  $YN$  interaction at finite momenta.

Thus, we come to the conclusion that the coincident observables considered above can be useful to help determine the lambda–nucleus optical potential at finite momentum.

## 4. Conclusions

In this paper we calculated the momentum dependences of the absolute differential cross sections for the production of  $\Lambda$  hyperons in coincidence with the  $K^+$  mesons from  $pA$  ( $A=\text{C, Cu, Ag, and Au}$ ) collisions at 2.83 GeV beam energy in the kinematical conditions of the ANKE experiment, performed at COSY, by considering incoherent primary proton–nucleon, secondary pion–nucleon  $\Lambda K^+$  production processes and processes associated with the creation of intermediate  $\Sigma^0 K^+$  pairs in the framework of a nuclear spectral function approach within the different scenarios for the  $\Lambda$  hyperon mean-field potential. It was found that the shapes of these cross sections are appreciably sensitive to the  $\Lambda$ –nucleus optical potential at lambda momenta less than 1.8 GeV/c. The above opens a good possibility to determine this potential here from the direct comparison of the presented in the work results with the data from the ANKE-at-COSY experiment. It was also demonstrated that the two-step pion–nucleon production channels dominate in the low-momentum  $\Lambda K^+$  creation in the chosen kinematics and, hence, they should be taken into account in the analysis of these data with the purpose of getting definite information on the lambda mean-field nuclear potential at finite momentum.

## Acknowledgments

The authors gratefully acknowledge A. Polyanskiy for his interest in this work, which has been partially financed by the Ministry of Education and Science of the Russian Federation.

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<sup>13)</sup>It should be noted that an analogous possibility was recently realized for the  $\omega$  mesons in [69].

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